SHARP ESTIMATES OF THE MEAN FIELD EQUATIONS AT CRITICAL PARAMETERS

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(joint work with Chang-Shou Lin)

Abstract: In this talk, we consider the following mean field equation

$$\Delta u(x) + \rho\left(\frac{h(x)e^{u(x)}}{\int_M h(x)e^{u(x)}dx} - \frac{1}{|M|}\right) = 4\pi\sum_{j=1}^{d}\alpha_j\left(\delta_{q_j} - \frac{1}{|M|}\right) \text{ in } M$$

where $\alpha_j \in \mathbb{N}$, δ_{q_j} is the Dirac measure at q_j and $\rho \in \mathbb{R}^+$ and (M, g) is a compact Riemann surface and $|M|$ is the area. Here Δ stands the Beltrami-Laplacian operator on (M, g). Let (u_k, ρ_k) be a sequence of bubbling solutions. Let p be a blow-up point of u_k, and $r > 0$ such that in $B_{2r}(p) \setminus \{p\}$, u_k has no blow-up points. We put

$$\rho_{k,p} = \frac{\rho_k \int_{B_r(p)} h(x)e^{u_k}dx}{\int_\Omega h(x)e^{u_k}dx} \text{ and } \rho_{\infty,p} = \lim_{k\to\infty}\rho_{k,p}.$$

Suppose $p \in \{q_1, ..., q_d\}$, that is, p is one of the vortex points. We derive the sharp estimate of the difference $(\rho_{k,p} - \rho_{\infty,p})$.

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