Research results

Knots, polynomials and isolated singularities

Knots and links can arise as the intersection of the vanishing set $V_f = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : f(x_1,x_2,x_3,x_4) = (0,0)\}$ of a polynomial map $f = (f_1, f_2) : \mathbb{R}^4 \to \mathbb{R}^2$ with a three-sphere $S^3 \subset \mathbb{R}^4$.

In joint work with Mark Dennis I developed an algorithm that constructs such polynomials for any given knot or link [2], a procedure that is useful for the creation of knotted configurations in physical systems [1]. The constructed polynomials can be taken to be polynomials in complex variables u and v and $\overline{v}$, a property that we call *semiholomorphic*.

The explicit construction allows upper bounds on the polynomial degree of f and knot invariants, such as the Morse-Novikov number of the link $L = V_f \cap S^3$, in terms of numbers that can be easily calculated from a braid word of a braid that closes to L [2].

It is not known for which links such polynomials can be taken to have an isolated singularity, i.e. which links are *real algebraic*. In [3] I show that links that are closures of squares of homogeneous braids are real algebraic. I proved this theorem by explicitly constructing polynomials of the desired form. It can then be shown that (as in the complex case) $f/|f| : S^3 \backslash L \to S^1$ is a fibration of the link complement.

Crossing numbers of composite knots

It is one of the oldest conjectures in knot theory that the minimal crossing number of a knot is additive under the connected sum operation. In [4], I established relations (in particular certain inequalities) between the crossing numbers of knots K_1, $K2$, certain spatial graphs and the crossing number of the composite $K_1 \# K_2$.

References

[1] B Bode, MR Dennis, D Foster, RP King. Knotted fields and explicit fibrations for lemniscate knots. *Proc R Soc A*. **475** (2017), 20160829.

[2] B Bode, MR Dennis. Constructing a polynomial whose nodal set is any prescribed knot or link. *arXiv:1612.06328.*

[3] B Bode. Constructing links of isolated singularities of real polynomials $\mathbb{R}^4 \to \mathbb{R}^2$. *arXiv:1705.09255.*

[4] B Bode. Crossing numbers of composite knots and spatial graphs. *Topology and its Applications*. **243** (2018), 33–51.