RESEARCH ACHIEVEMENTS

SUMMARY OF THE MAIN RESEARCH RESULTS PUBLISHED SINCE 2009

1. *A proof of the simultaneous blow-up for a system of PDEs describing chemotaxis.*
   I proved that radial solutions for a two special Keller-Segel model has in general simultaneous blow-up. Biologically this phenomenon show a remarkable phenomenos of self-organization among the cells under aggregation by chemotaxis. Research results published in papers [10, 5].
   Mathematical tools: Asymptotic analysis, semigroups theory.

2. *Extension of the threshold number for the parabolic-elliptic Keller-Segel model to a sharp curve when having two different kind of cells.*
   Through the use of several tools of variational analysis, I got to generalize the known theory of global existence and blow-up to the case of having many kinds of cells. This achievement was sharp. I got this result in the set of papers [6, 8, 5].
   Mathematical tools: Variational analysis, Lyapunov functions, entropy methods.

3. *Description of Keller-Segel models having a mortality rate.*
   In this research project, I got a new version of Moser-Trudinger inequality for case of having singularities. I applied this result to prove the existence of global in time solutions for a system of partial differential equations when the source term has singularities of the delta dirac type. This research was published in the paper [2].
   Mathematical tools: Variational analysis, variation of parameters, entropy methods, blow-up techniques.

4. *Improvement of several results of global existence for drift-dffusion systems.*
   My work consisted into find a characterization of the conditions for having global existence or blow-up for a set of drift-diffusion systems. Research results published in papers [7, 5, 9]
   Mathematical tools: Lyapunov function for parabolic equations, entropy methods,variational analysis, Moser-Alikakos iteration, Radon-Nikodym theorem.

5. *Blowup in higher dimensional two species chemotactic systems.*
   I got an extension for a family of generalized Keller-Segel models in high dimensions. The main novelty is that we do not need radial symmetry assumption on the solutions as previously was being assume in the literature. Research published in paper [4].
   Mathematical tools: Moment blow-up technique, Orlicz spaces.

6. *Proposal of a new mathematical model describing the dynamics of fertilization of corals.*
   I created a mathematical model to analyze the rate of fertilization of corals. I got to prove mathematically that chemotaxis can improve significatively the rate of fertilization. I also found enough conditions on the initial data that allow us to conclude a succesul fertilization when the time is large. This research was developed in papers [12,13 and 14].
   Mathematical tools: semigroups theory, Navier-Stokes theory, renormalized solutions of partial differential equations.

1. Espejo Elio, Suzuki Takashi. *Reaction Enhancement by Chemotaxis,* Nonlinear Analysis: Real world applications, 35, 102–131, 2017.

2. Espejo Elio, Suzuki, Takashi. *Reactions terms avoiding aggregation in slow fluids.* Nonlinear Analysis: Real World Applications. 21, 110-126, 2015.

3. Elio Espejo, Takashi Suzuki. *Global existence and blow-up for a system describing the aggregation of microglia.* Applied Mathematics Letters 35C, 2014 , 29-34.

4. Espejo Arenas, Elio Eduardo; Wolansky Gershon. *The Patlak–Keller–Segel model of chemotaxis on $\mathbb{R}^2$ with singular drift and mortality rate.* Nonlinearity 26, 2315–2331, 2013.

5. Espejo Arenas, Elio Eduardo; Masaki Kurokiba; Suzuki, Takashi. *Blowup threshold and collapse mass separation for a drift-diffusion system in-space dimension two.* Communications in pure and applied analysis (CPAA). Volume 12, issue 6, 2627-2644, 2013.

6. Elio Espejo, Karina Vilches and Carlos Conca. *Sharp condition for blow-up and global existence in a two species chemotactic Keller–Segel system in* $\mathbb{R}^2$. European Journal of Applied Mathematics, 24, pp 297-313, 2013.

7. Biler, Piotr; Espejo, Elio; Guerra, Ignacio. *Blowup in higher dimensional two species chemotactic systems.* Communications in pure and applied analysis (CPAA). Volume 12, issue 1, 2013.

8. Espejo Arenas, Elio Eduardo; Stevens Angela; Suzuki, Takashi. *Simultaneous blowup and mass separation during collapse in an interacting system of chemotactic species.* Differential and integral equations. Volume 25, numbers 3-4 (2012), 251-288.

9. Conca, Carlos, Espejo Elio. *Threshold condition for global existence and blow-up to a radially symmetric drift-diffusion system.* Applied Mathematics Letters 25 (2012) 352-356.

10. Conca, Carlos, Espejo Elio, Karina Vilches. *Remarks on the blowup and global existence for a two species chemotactic Keller-Segel system in* R^2. European Journal of Applied Mathematics (2011), vol. 22, pp. 553-580.

11. Espejo Arenas, Elio Eduardo, Angela Stevens, Juan Velázquez. *A Note on Non-Simultaneous Blow-up for a Drift-Diffusion Model.* Differential and Integral Equations Volume 23, Numbers 5-6 (2010), 451-462. ISSN 0893-4983

12. Espejo Arenas, Elio Eduardo, Angela Stevens, Juan Velázquez. *Simultaneous Finite Time Blow-up in a Two-Species Model for Chemotaxis.* Analysis (2009), Vol. 29, Issue 3, 317–338. ISSN 0174-4747

RESEARCH GRANTS ACQUIRED

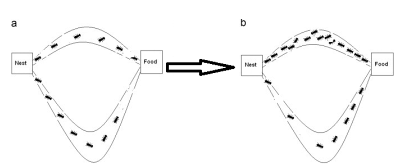
- US$50000 research grants given by the colombian goverment to support during the years 2015-2016 the research project "*Global existence and qualitative behavior for a mathematical model describing the fertlization of corals*".

RESEARCH PLAN

Mathematical analysis of emergence systems:

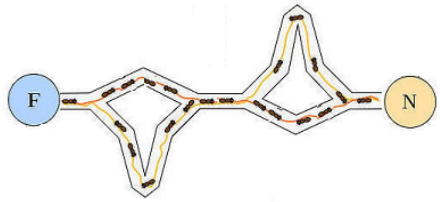
Mathematical analysis of networks in biology

Emergence makes reference to the collective behavior of organisms which does not arise when they are alone. An interesting example is the *ant trail formation.* During the ant foraging it is known that ants follow simple behavioral rules based on local information. What makes it interesting is that this simple behavior produce complex organized and seemingly intelligent strategies at the population level. It has been observe that ant colonies can even find the shortest path between their nest and a food source. One important feature during this process is the capacity of pheromones to evaporate. Since the moment the ants leave the nest, they keep sending a chemical signal. Once they find the food source, they go back to the nest by following the same chemical signal and at the same time reinforcing it. As the time progresses, the pheromone on non-optimal paths evaporate while the pheromone on near-optimal paths is reinforced. Next figure illustrates this process.



Even in the case that ants move through a maze, it has been shown that they can find the shortest path between the food source and the nest. Next figure illustrates one instance.

Let us consider the dynamics of ants in the presence of n nests. Let us suppose that the ants keep the food inside those nest, in such a way that they are transporting permanently food among the different nest to guarantee the



survival of the whole community. This kind of behavior, has already been observe in the nature with a kind of ant named *Argentine ant* or *Linepithema humile* (cf. [9]). What is specially remarkable about all this dynamic is that the Argentine ant colony is able to find the path among all the nests that minimize the required energy to connect the whole net. In other words, the ants solve a mathematical problem known in combinatorial optimization as the Steiner problem. More precisely the statement of the Steiner problem is the next: *Given N points in the plane, we must connect them by lines of minimum total length in such a way that any two points may be interconnected by line segments either directly or via other points and line segments.*

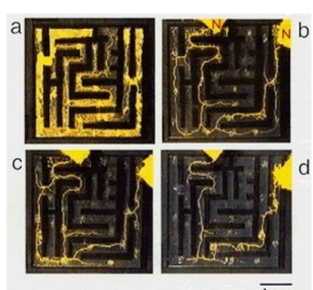
Let us now suppose that among all the ant's nest, we remove one. The colony of ants have showed a remarkable capacity of adaptation being able to construct efficiently a new optimal network. Having a mathematical model at hand describing this dynamics would open new approaches for solving problems in engineer dealing with networks. One remarkable example is the Mobile ad-hoc networks (MANETs). These are special kind of wireless mobile nodes which form a temporary network without using a centralized administration. It is a challenging task to find most efficient routing mainly due to two reasons:

- the changing topology, and
- the dynamic behavior of the nodes in MANET.

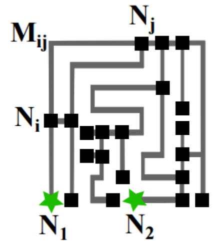
A mathematical model describing the dynamics of ants when moving among nests would be suitable for finding the adaptive routing for such type of network.

Main project goal: *Construct and analyze mathematical models showing the capability of different organisms of finding the shortest path between two or more different points and to look for its applications in engineer.*

Several models of nonlinear differential equatios have been proposed in the literature to describe the dynamics of ants (cf. [3],[4]). However, typically these models do not show the shortest path asymptotically. This feature makes difficult its implementation to solve networks problems in engineer (cf. [8]). To the best of my knowledge there is until now only one mathematical model using differential equations being able to show asymptotically the ability of some organisms to find shortest path on a network. This mathematical model was developed for describing the dynamics of a type of amoeba named Physarum Polycephalum (cf. [10, Tero et al]). When harvesting, it spreads all around looking for food sources. After a certain length of time this amoeba will connect all the found food sources in a optimal way. Next picture shows the Physarum (in yellow) initially distributed uniformly on a maze and how it concentrates exactly on the shortest path joining two food sources.



In order to construct this model *Tero et al* introduce a graph like the next one.



The nodes for the food sources and named N_1 and N_2, the other sources are denote N_3, N_4 and so forth. The section between the node N_i and N_j is denote by M_{ij} and its length is denoted by $|M_{ij}| = L_{ij}$. The presure at the nodes N_i and N_j are denoted by p_i and p_j. It is assumed that the flux Q_{ij} between N_i and N_j satisfies the Pousille law for gases, in consequence one have

$$Q_{ij} = \frac{D_{ij}}{L_{ij}}(p_i - p_j) \tag{1}$$

where D_{ij} represents its conductivity and it is given by

$$D_{ij} = \frac{\pi r_{ij}^4}{8\eta} \quad \text{for all } i, j = 1, 2, \ldots$$

where r_{ij} is the radius of the cylinder joining N_i and N_j and η is the viscosity of the fluid. The coefficients of pressure p_i are determined by mean of the Kirschhoff's law. In order to explain it briefly, let I_0 be the flux flowing from the first food source and by $-I_0$ the flux flowing from the second food source. As the amount of fluid must be conserved, it is assumed zero capacity at each node, then

$$\sum\nolimits_i Q_{i1} + I_0 = 0,$$

$$\sum\nolimits_i Q_{i2} - I_0 = 0$$

and

$$\sum\nolimits_i Q_{ij} = 0, \quad \text{for } j \neq 1, 2.$$

The pipe-flow analogy (cf. (1)) essentialy says that the mass transport is directly proportional to the pressure differences and the thickness of the tube. Then it is proposed to construct a mathematical model having two basic caracteristics:

1. The conductivity tends to decay exponentially in the absence of flux.
2. The presence of a flux will enhanced the conductivity.

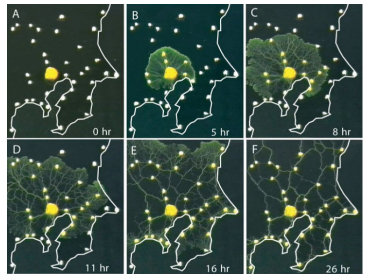
In other words, high rates of streaming stimulate an increase in tube diameter, whereas tubes tend to dissapear at low flow rates. The proposed model having this properties is

$$\frac{dD_{ij}}{dt} = f(|Q_{ij}|) - aD_{ij} \tag{2}$$

where f is *any* increasing function with $f(0) = 0$. Different numerical simulations for different choices of f have shown an outstanding results for solving the shortest path.

The capacity of the Physarum to construct optimal two dimensional networks

Scientist placed different food sources for the Physarum on a surface in a location that corresponded to the cities surrounded Tokyo (cf. [11]), and allowed the Physarum to grow outwards from the center. Then it was noticed how the Physarum self organized in a network that was comparable in efficiency to real Tokyo's train network.



Physarum connecting the train stations in a map of Tokyo.

In order to describe this dynamics matematically, it was proposed an adaptation of the aproach to describe the dynamics of the physarum on a maze, given by equation (2). The initial shape of the physarum is represented by a randomly and densely meshed latice. At each time step, two food sources are selected randomly.

Specific research goals: Taking into account the different experiments made with ants and bacteria like the Physarum, I proposed to attack next problems.

1. Construct mathematical models explaining how optimal networks arise in the nature.
2. Make a mathematical and numerical analysis of the proposed models
3. Look for different applications in engineer of these models.

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