(2) Abstract of results

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(a) **On a relation between a polytope duality for families of $K3$ surfaces and coupling.**

Introduced by Ebeling, coupling is a duality between two weight systems using the magic square. Bisides, for some weight systems that are coupling pairs, one can find a pair of defining invertible polynomials of some singularities in $\mathbb{C}^3$ to be strange-dual, and then, they can be projectivized as anticanonical sections in the weighted projective spaces with weight systems determining simple $K3$ singularities.

Take defining polynomials f and f' of strange-dual singularities in $\mathbb{C}^3$ and their invertible projectivizations F and F' respectively as anticanonical sections in the weighted projective spaces $\mathbb{P}_a$ and $\mathbb{P}_b$ of weights a and b. We concluded that almost all the strange-dual pairs admit an extension of the Newton polytopes to polar-dual polytopes. We also expect that the result is applied to the geometrical study of reflexive polytopes associated to families of $K3$ surfaces.

(b) **On integral lattices with an automorphism and isolated hypersurface singularities.** (joint work with Professor Claus Hertling)

A pair (H, h) of a finitely-generated $\mathbb{Z}$-lattice H and an automorphism h on it is said to admit a standard decomposition into Orlik blocks if the following two conditions are satisfied:
(1) H admits a decomposition into h-invariant sublattices $H^{(i)}$ for which there exists an element $e_0^{(i)}$ such that $H^{(i)}$ is generated by finitely-many orbits of $e_0^{(i)}$ by h.
(2) The characteristic polynomials of h on $H^{(i)}$, which we denote by $p_{H^{(i)},h}$, divides the characteristic polynomial $p_{H^{(i-1)},h}$ for $H^{(i-1)}$, for all i.
In other words, the lattice H admits a decomposition with elementary divisors being its characteristic polynomials. In 1972, Orlik conjectured that for any isolated hypersurface singularities, its Milnor lattice together with a monodromy action admit a standard decomposition into Orlik blocks. In our study, we consider Orlik's conjecture for several cases.

Part 1: Let (H, h) be a pair of a single Orlik block H and an automorphism h and consider the power (H, h^μ) of the lattice, where μ is the rank of H. Firstly, we give a sufficient condition for (H, h^μ) to admit a standard decomposition into Orlik blocks in terms of the set of orders of eigenvalues of h. As an application, together with a result by Orlik and Randell in 1977 that the Milnor lattice of chain-type singularity is of the form $(H_{Mil}, h_{Mil}) = (H_{Mil}, h^\mu)$ with some automorphism h, we can give a sufficient condition for (H_{Mil}, h_{Mil}) to admit a standard decomposition into Orlik blocks.

Part 2: Consider two single Orlik blocks $(H^{(1)}, h^{(1)})$ and $(H^{(2)}, h^{(2)})$ and their tensor product $(H^{(1)} \otimes H^{(2)}, h^{(1)} \otimes h^{(2)})$. Secondly, we give a sufficient condition for $(H^{(1)} \otimes H^{(2)}, h^{(1)} \otimes h^{(2)})$ to admit a standard decomposition into Orlik blocks in terms of the set of orders of eigenvalues of $h^{(1)} \otimes h^{(2)}$. As an application, we obtain a method to judge a Thom-Sebastiani sum of two appropriate singularities admits a standard decomposition into Orlik blocks.

Part 3: Thirdly, we introduce a notion of "excellent orders" and partly interprete above results.

Part 4: We consider whether or not the Milnor lattice together with a monodromy action admits a standard decomposition into Orlik blocks for cyclic-type singularities. Cooper studied this topic in 1982, but it contains some errors. Following his method using spectral sequences, we carefully investigate generators of the lattice and finally obtain a standard decomposition of the Milnor lattice into Orlik blocks.