Research Achievements

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In my research I introduced the concept of "minimal submanifolds with symmetries", such as *weakly reflective submanifolds* (Ikawa-Sakai-Tasaki 2009), into a class of *proper Fredholm (PF) submanifolds in Hilbert spaces* (Terng 1989) and showed that there exist many infinite dimensional minimal PF submanifolds with symmetries in Hilbert spaces. Let G/K be a compact normal homogeneous space, $\mathfrak{g}$ the Lie algebra of G and $V_{\mathfrak{g}} := L^2([0,1],\mathfrak{g})$ the Hilbert space of all L^2-paths from the interval $[0,1]$ to $\mathfrak{g}$. Then an infinite dimensional Riemannian submersion $\Phi : V_{\mathfrak{g}} \to G/K$ called the *parallel transport map* (Terng-Thorbergsson 1995) is defined so that for any submanifold N of G/K the inverse image $\Phi^{-1}(N)$ is a PF submanifold of $V_{\mathfrak{g}}$. In the following, I introduce my main research achievements.

• On weakly reflective PF submanifolds in Hilbert spaces ([2] in the list)

In this paper, first I extended the concept of weakly reflective submanifolds to a class of PF submanifolds in Hilbert spaces. Next I showed that each fiber of the parallel transport map $\Phi : V_{\mathfrak{g}} \to G/K$ is a weakly reflective PF submanifold of the Hilbert space $V_{\mathfrak{g}}$. Then supposing that G/K is a Riemannian symmetric space of compact type I showed that for any weakly reflective submanifold N of G/K the inverse image $\Phi^{-1}(N)$ is a weakly reflective PF submanifold of $V_{\mathfrak{g}}$. Moreover, applying these results to examples of weakly reflective submanifolds in G/K I constructed many examples of weakly reflective PF submanifolds of $V_{\mathfrak{g}}$. Furthermore I computed the shape operator of the PF submanifold $\Phi^{-1}(N)$ under the most general setting, and showed a necessary and sufficient condition for $\Phi^{-1}(N)$ to be totally geodesic. Using this I showed that so obtained weakly reflective PF submanifolds are not totally geodesic (therefore not reflective). As a consequence it turns out that in infinite dimensional Hilbert spaces there exist many homogeneous minimal submanifolds which are not totally geodesic, unlike in the finite dimensional Euclidean case.

• Austere and arid properties for PF submanifolds in Hilbert spaces ([3] in the list)

In this paper I studied the similar problem of [2] for *austere submanifolds* (Harvey-Lawson 1982) and *arid submanifolds* (Taketomi 2018). Note that these two submanifolds generalize weakly reflective submanifolds. First I computed the principal curvature of a PF submanifold $\Phi^{-1}(N)$. Although such a computation has already been studied by Koike, I gave a different elementary proof by using the formula for the shape operator given in [2]. Next I studied the auster property and showed that if G/K is a sphere then the austere property of N is equivalent to the austere property of $\Phi^{-1}(N)$. Applying this result to the classification result of austere orbits in s-representations (Ikawa-Sakai-Tasaki 2009) I constructed examples of austere PF submanifolds in Hilbert spaces. Then I studied the arid property and showed that if N is an arid submanifold of a symmetric space G/K of compact type then the inverse image $\Phi^{-1}(N)$ is an arid PF submanifold of $V_{\mathfrak{g}}$. Applying this result to examples of arid submanifolds in G/K I showed examples of arid PF submanifolds of $V_{\mathfrak{g}}$. In particular, from an example of arid submanifold of the sphere which is not austere (Taketomi 2018) I constructed an example of an arid PF submanifold which are not austere. From these result we see that there exist many (homogeneous) minimal PF submanifolds with symmetries in Hilbert spaces. Thus it is a problem to classify homogeneous minimal submanifolds in Hilbert spaces. I proposed this problem and mentioned the relation to affine Kac-Moody symmetric (Terng 1995, Heintze 2007).

• On weakly reflective submanifolds in compact isotropy irreducible Riemannian homogeneous spaces ([4] in the list)

In this paper I extend the main results of the papers [2] and [3] to the case that G/K is a compact isotropy irreducible Riemannian homogeneous space (which is not necessarily symmetric). Moreover I characterized so obtained weakly reflective PF submanifolds $\Phi^{-1}(N)$ and showed a necessary and sufficient condition for a submanifold N of G/K to be weakly reflective. Note that in this paper the notion of $P(G,H)$-actions, where G is usually assumed to be connected, is extended to the case that G is not necessarily connected. The main result of this paper is based on the discussion with Professors Ernst Heintze and Takashi Sakai during my visit to the University of Augsburg.