研究計画書（英訳）

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So far I have studied several complex variables. In particular, I have achieved results on the automorphism groups of domains in a complex Euclidean space. There are some unresolved problems in the research. Among them, I will describe two goals for the future. In addition, I will describe one goal which I was working on until recently.

Study on discrete subgroups of automorphism groups

In [2], [3] we considered the following domain:

$$D^{p,q} = \{(z_1, \ldots, z_{p+q}) \in \mathbb{C}^{p+q} : -|z_1|^2 \cdots - |z_q|^2 + |z_{q+1}|^2 + \cdots + |z_{p+q}|^2 > 0\}$$

This domain is a homogeneous domain where the automorphism group acts transitively. In [2], [3], we determined the automorphism group $\mathrm{Aut}(D^{p,q})$. Then, we showed that any quotient space of $D^{p,q}$, for $p > q$, is non-compact. On the other hand, for $q \geq p$, the properties of discrete subgroups of $\mathrm{Aut}(D^{p,q})$ have not been studied. I want to clarify this point. The expected conclusion is that, like $p > q$, the discrete subgroups are very small and any quotient space is not a compact space. We are looking for a method to show this.

Study on the Lie group structure of automorphism groups and core

We denote by $\mathrm{Aut}(M)$ the automorphism group of a complex manifold M. $\mathrm{Aut}(M)$ is not a Lie group in general. For example, for $M = \mathbb{C}^n$, $n > 1$, $\mathrm{Aut}(M)$ is a huge topological group and not a manifold. On the other hand, if M is a bounded domain, $\mathrm{Aut}(M)$ always has a Lie group structure. Therefore, the question is when an automorphism group have a Lie group structure. But it is not completely understood, particular in the case of unbounded domains. We showed in [4] that the automorphism groups of, the most important classes, strong pseudoconvex domains and finite type domains have Lie group structures. This phenomenon is shown using Bergman pseudometrics. However, the problem is not completely solved by this method, and generalization to complex manifolds is still unsolved.

In [4], we showed that a study of subsets $\mathfrak{c}'(\Omega) \subset \Omega$ of Ω, called the core, is related to the above problem. The research of the core has been studied by people at the University of Wuppertal with a different motive than what is written here, but much is still unknown. Also I want to study the property of the core.

Study on Hartogs type extension theorem of holomorphic functions

This subject has nothing to do with the study of automorphism groups, but I have recently studied a variation of Hartogs' theorem in [5], which was the starting point of the theory of several complex variables. Hartogs' theorem says that if we consider a domain Ω and a compact set K in a higher dimensional complex Euclidean space, and if $\Omega \setminus K$ is connected, then any holomorphic function on $\Omega \setminus K$ extends to Ω. Roughly speaking, any holomorphic function on a neighborhood of the boundary of the domain extends to the whole domain. Here, for a holomorphic function near a subset of the boundary, it cannot extend any more in general. However, if we add a certain convexity condition to the subset , the holomorphic function extends to the whole domain, like Hartogs' theorem. For the convexity conditions described in the paper [5], there are still some points that need to be improved. I want to do this through a joint research.