

Futher works

In our previous works, we studied

1. Mathematical analysis of reaction-diffusion systems with control of total mass and entropy dissipation.
2. Mathematical analysis of a semilinear heat equation with the nonlinearity of type $|u|^{p-1}u$.

We continue studying above topics as follows:

Mathematical analysis of reaction-diffusion systems describing reversible chemical reactions.

1. Asymptotic behavior of the solution to reaction-diffusion systems with inhomogeneous Dirichlet boundary condition.

In our previous works [1], we proved that the solution to chemical reaction-diffusion systems under inhomogeneous Dirichlet boundary conditions which correspond to an equilibrium state of reaction terms. In our future works, we consider the case that the boundary condition is apart from equilibrium state. In this case, difficulty appears because of the defect of the control of total entropy. We first study the existence of the solutions to stationary problem and then, analyze the time evolution of the solution when the initial data is close to the stationary solutions. We expect stability of the solution is obtained in this case. Finally, we consider the case the initial data is far from the stationary solutions.

2. Asymptotic behavior of the solution to reaction-diffusion systems with nonlinear diffusion.

We consider the asymptotic behavior of the solution to chemical reaction-diffusion system with reversible reaction and nonlinear diffusion. It is known that, under the assumption that the speed of the diffusions are “sufficiently slow”, exponential convergence toward the equilibrium is proved (Fellner-Latos-Tang). In our future works, we generalize our previous work [2] to the case that diffusion term is nonlinear and try to weaken the assumption for the diffusion speed.

3. Global existence of the solution to reaction-diffusion systems with strong nonlinearity of the diffusive term.

In our previous work [3], we considered reaction-diffusion systems with nonlinear diffusion (porous media type $-\Delta u^m$) and proved global existence in time of the solution under the assumption that $m \leq 2$. We try to weaken this assumption in the future works and also, we consider the global existence of renormalized solutions.

Local in time well-posedness of the solution to a semilinear heat equation with singular initial data

In our previous work [4], We considered a semilinear heat equation with the nonlinearity of type $|u|^{p-1}u$ and proved local in time existence of the solution with the initial data in negative Sobolev spaces. This result is a generalization of the “sub-critical case” with initial data in L^q . On the other hand, the result which corresponds to “critical case” is still an open problem and we will consider this problem in the future works.