RESEARCH RESULTS

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My research interests are representation theory of Lie algebras and applications to related mathematics and mathematical physics. Especially, recently I am interested in the $\imath$quantum groups, which are algebraic systems appearing in the theory of quantum symmetric pairs. A quantum symmetric pair is a quantization of certain pair of complex Lie algebras. To be more specific, it is a pair of a Drinfeld-Jimbo quantum group and its certain coideal subalgebra. Such a subalgebra is referred to as an $\imath$quantum group. Based on an idea "The $\imath$quantum groups are generalizations of the quantum groups"($\imath$program), I aim to generalize important results in the theory of quantum groups to the $\imath$quantum groups setting.

(1) **Representation theory of the $\imath$quantum group of type AIII.**

Bao-Wang proved that the $\imath$quantum group of type AIII and the Hecke algebra of type B(=C) are in Schur duality. In joint works with them, I extended this result to the unequal parameter case. In particular, this extension covers the asymptotic case. Applying the representation theory of the Hecke algebra of type B with asymptotic parameter via Schur duality, I described the structures of finite-dimensional irreducible modules over the $\imath$quantum group of type AIII in detail.

Also, by extending the theory of $\imath$canonical bases due to Bao-Wang, they and I determined the irreducible characters of the ortho-symplectic super Lie algebra completely.

(2) **Classical weight modules.**

I defined the notion of classical weight modules in representation theory of the $\imath$quantum groups by generalizing the notion of weight modules in representation theory of the quantum groups. I showed that the classical weight modules form a good class in finite-dimensional representation theory of the $\imath$quantum groups. I classified the finite-dimensional classical weight module for the $\imath$quantum groups of type AI, AII, and AIII.

(3) **Representation theory of the $\imath$quantum group of type AIII and combinatorial structures.**

By analyzing the module structures of finite-dimensional irreducible classical weight modules over the $\imath$quantum group of type AI in detail, I proved that they admit based module structures. This result is a generalization of the theory of canonical bases in representation theory of the quantum groups. However, it differs from the theory of $\imath$canonical bases, which is also a generalization of canonical bases. Furthermore, I obtained a combinatorial structure which reflects the module structures of finite-dimensional classical weight modules sufficiently. This result is different from $\imath$canonical bases, too.

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