Research Result

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• tensor model

The tensor model can be understood as a generalization of rectangular matrix model to higher rank. Recently, the tensor model is receiving a lot of attention because of its relation to the low dimensional AdS/CFT and quantum gravity. However, it is much more difficult to clarify the nature of the tensor model than the matrix models. For example, the set of the operators of the tensor model is nontrivial unlike matrix model.

In order to resolve the enumeration problem of the operators, I focused on the so-called cut operation. The cut operation corresponds to the variation of the measure of the partition function under change of variables. I extended the cut operation to "generalized cut" that includes higher-order contributions. By selecting a variation function, various operators can be generated by the generalized cut operation. I also arrived at a conjecture about the selection of appropriate variation function to generate all operators. In the case of the rank 3 model, it was confirmed that the conjecture is correct at least within the region examined.

I demonstrated that Op/FD correspondence is established between tensor models of different ranks. Here the Op/FD correspondence is a one-to-one correspondence between operators in the rank r model and Feynman diagrams in the rank $r-1$ model . All operators in the tensor model are, therefore, labeled with the Feynman diagram. In particular, in the case of rank 3, it is extended to an Op/FD/dessin correspondence including a one-to-one correspondence with graphs called dessins. The dessin is a graph consisting of vertices of two colors and edges connecting them embedded on a two-dimensional surface. I succeeded in building a concrete relationship. The cut & join operations defined on the operators of the tensor model can be interpreted as those on the dessins by using the Op/dessin correspondence. The join & cut operations which are rather complicated on the operators become easier to handle by adding the interpretation as a geometric operation on the dessins.

• 2d/4d(5d) connection

The 2d/4d connection states the equivalence between the conformal block in 2d CFT and the instanton partition function in 4d supersymmetric gauge theory. The connection between the q-W_n conformal block and the 5d instanton partition function was also proposed. These have two parameters q and t. By taking the limit $q, t \to 1$, this connection reduces to the 2d-4d one. I have studied the root of unity limit in q and t. The generators of the q-Virasoro (q-W_2) algebra can be described by a q-boson field. In the $q, t \to -1$ limit, the generators of the superconformal algebra appear. Similarly, the free boson and free fermion which describe this algebra can be obtained from the q-boson in this limit. On the 5d side, I obtained the 5d instanton partition function in the root of unity limit which is the same as that used on the 2d side. I have confirmed that the results are equal to the 4d ALE instanton partition function at the lower level at least. The 2d-4d connection can be understood through the limiting procedure in the 2d/5d connection.

In the general r-th root of unity limit of q-W_n algebra, the $\mathbf{Z}_r$-parafermions appear. The obtained theory is the coset CFT which has $\frac{\widehat{sl}(n)_r \oplus \widehat{sl}(n)_p}{\widehat{sl}(n)_{r+p}}$ symmetry. In fact, the central charge of the energy-momentum tensor is exactly reproduced. The parameter p is related with the omega-background of the corresponding gauge theory and its relation was also clarified.

I considered also another root of unity limit ($q \to 1,\ t \to -1$). There exists the correspondence between a modified affine $sl(2)_k$ current block and the instanton partition function in the presence of a surface operator. The representation of affine $sl(2)_k$ algebra can be realized in terms of free fields which are obtained from the q-boson at the root of unity limit as mentioned above. I presented explicitly the free field representation of affine $sl(2)_k$ algebra and derived the integral representation of the modified current blocks.