## Research results

Let $p$ be an odd prime. Triple product $L$-functions attached to triplets of modular forms have Euler products and functional equations. We have studied $p$ adic triple product $L^{-}$ functions which are $p$-adic analytic functions interpolating an infinite number of central critical values of triple product $L$-functions attached to triplet of modular forms.

## Proceeding research

Let $f$ be an ordinary cusp form. Haruzo Hida constructed a Hida family $F$ attached to $f$ which was a $p$-adic family of ordinary cusp forms. The Hida family $F$ which interpolates $f$ is unique. Let ( $f, g, h$ ) be a triple of ordinary cusp forms. Then there exists the triple ( $F, G, H$ ) of Hida families attached to ( $f, g, h$ ). Ming-Lun Hsieh constructed a $p$-adic triple product $L$-function attached to ( $F, G, H$ ) in [Hsi17]. As a generalization of Hida family, Colman families were defined in [Corollary B5.7.1, Col97]. Let $f$ be a cusp form which is not ordinary. Then we can construct a Coleman family $F$ which interpolates $f$ uniquely.

## Main results

Let ( $f, g, h$ ) be a triple of cusp form of weight $(k, l, m)$. If there exists a triangle with sides $(k, l, m)$, we say that the triple ( $f, g, h$ ) satisfies a balanced condition. Otherwise, we say that the triple ( $f, g, h$ ) satisfies an unbalanced condition. If a $p$-adic triple product $L^{-}$ function interpolates central critical values of triple product $L$-functions attached to triples of cusp forms satisfing the balanced condition (resp. unbalanced condition), the $p$-adic tripe product $L$-function is called the balanced (resp. unbalanced) $p$-adic triple product $L$-function. In [Fuk19], we generalized his results [Hsi17] under unbalanced $p$ adic triple product $L$-functions. Let $F$ be a Hida family and $G, H$ general $p$ adic families of cusp forms. We constructed a unbalanced $p$-adic triple product $L$-function attached to ( $F, G, H$ ). In the previous research [Hsi17], he constructed unbalance $p$-adic triple product $L$-functions attached to triples of Hida families. In our result, we can take more general $p$-adic families for $G, H$. For example, we can take Hida families, Colemna families or CM-families as an exmples of $G, H$.

## Reference

[Col97] R. F. Coleman, p-adic Banach spaces and families of modular forms. Invent. Math., 127(3):417-479, 1997.
[Fuk19] K. Fukunaga, Triple product p-adic L-function attached to p -adic families of modular forms, arxiv:1909.03165.
[Hsi17] M.-L. Hsieh, Hida families and p-adic triple product L-functions, AJM, to appear.

