TITLES AND ABSTRACTS

Geodesic orbit metrics in compact homogeneous manifolds with equivalent isotropy summands

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Abstract: A geodesic orbit manifold (g.o. manifold) is a Riemannian manifold (M, g) with the property that any geodesic in M is an orbit of a one-parameter subgroup of a group G of isometries of (M, g). The metric g is then called a G-g.o. metric in M. We investigate homogeneous geodesics in a class of homogeneous spaces called M-spaces, which are defined as follows. Let G/K be a generalized flag manifold with $K = C(S) = S \times K_1$, where S is a torus in a compact simple Lie group G and K_1 is the semisimple part of K. Then the *associated M-space* is the homogeneous space G/K_1. These spaces were introduced and studied by H.C. Wang in 1954. We prove that for various classes of M-spaces the only g.o. metric is the standard metric. For other classes of M-spaces we give either necessary, or necessary and sufficient conditions, so that a G-invariant metric on G/K_1 is a g.o. metric. The analysis is based on properties of the isotropy representation $\mathfrak{m} = \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s$ of the flag manifold G/K (as $\mathrm{Ad}(K)$-modules) and corresponding decomposition $\mathfrak{n} = \mathfrak{s} \oplus \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s$ of the tangent space of the M-space G/K_1 (as $\mathrm{Ad}(K_1)$-modules).