

Index for Ohashi Lecture series

Lecture I: Domain walls and Vortices

- (a) The simplest soliton: domain wall
- (b) Axially symmetric vortex in Abelian-Higgs theory
- (c) Asymptotic behaviors and inter-soliton forces
  - i. type I and type II vortex
- (d) Derrick's theorem

Lecture II: Supersymmetry and superfields

- (a) Supersymmetry algebra
- (b) chiral superfield
  - i. Kähler potential and Kähler trf.
  - ii. super potential
- (c) vector superfield
  - i. super gauge trf. and WZ gauge
  - ii. FI term
- (d) supersymmetric gauge-Higgs model
  - i.  $D$ -term and  $F$ -term conditions for vacua

Lecture III: Non-linear sigma model (NL $\sigma$ M) on Kähler manifold  $\mathcal{M}$

- (a) Strong coupling limit of SUSY gauge-Higgs theory
- (b) Target manifold  $\mathcal{M}$  and vacua in the Higgs phase
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  - i. Patches, transition functions and Kähler trf.
  - ii. Fubini-Study metric for  $\mathcal{M} = \mathbb{C}P^{N_f-1}$
  - iii. Baryonic fields  $B$  and Plücker relations
  - iv. Kähler potential  $K(B, B^\dagger)$
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- (d) Target space  $\mathcal{M}$  for  $G = SO(N), USp(2N)$ 
  - i. 'Meson'  $M$  as the  $G^{\mathbb{C}}$  invariant
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- (a) Discrete vacua with non-degenerate mass terms
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  - iv. Topological sector and cell decomposition of  $\mathbb{C}P^{N_f-1}$
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  - iii. domainwall solutions and string theory perspective

Lecture V: BPS vortices

- (a) A vacuum and BPS equations
- (b) The moduli matrix and constraints on the  $G^{\mathbb{C}}$  invariant
- (c) Semilocal vortex for  $G = U(1)$  and lump in NL $\sigma$ M on  $\mathbb{C}P^{N_f-1}$ 
  - i.  $\pi_2[\mathcal{M}]$  and lump solutions
  - ii. scale moduli and small lump singularity
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  - i. Local vortex with a strong condition and orientational moduli
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- (a) Hannany-Tong model from string theory perspective
  - i.  $D$ -term condition and a moduli space  $\mathcal{M}'_v$
- (b) Kähler quotient construction of the vortex moduli space
  - i. half-ADHM relation
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- (a) Composite solitons in 5D
  - i. BPS equations and instanton charge
  - ii. Moduli matrix
  - iii. Intersection of vortices
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- (a) Mass gap, higher derivative expansion and an effective action
- (b) Effective action on BPS domainwalls
  - i. Effective action formula in superspace
  - ii. Exact examples with the strong coupling limit
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