Generalized Lax-Milgram theorem in Banach spaces and its application to the elliptic system of boundary value problems.

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We generalize the well-known Lax-Milgram theorem on the Hilbert space to that on the Banach space. Suppose that $a(\cdot,\cdot)$ is a continuous bilinear form on the product $X \times Y$ of Banach spaces X and Y, where Y is reflexive. If null spaces N_X and N_Y associated with $a(\cdot,\cdot)$ have complements in X and in Y, respectively, and if $a(\cdot,\cdot)$ satisfies certain variational inequalities both in X and in Y, then for every $F \in N_Y^{\perp}$, i.e., $F \in Y^*$ with $F(\phi) = 0$ for all $\phi \in N_Y$, there exists at least one $u \in X$ such that $a(u, \varphi) = F(\varphi)$ holds for all $\varphi \in Y$ with $\|u\|_X \le C\|F\|_{Y^*}$. We apply our result to several existence theorems of L^r-solutions to the elliptic system of boundary value problems appearing in the fluid mechanics.