Competitive system of chemotaxis

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A competitve system of chemotax is introduced by Espejo, Stevens, and Velázquez. It is a generalization of the Smoluchowski-Poisson equation of single component, taking regards in the age structure of cells. Unexpectedly, simultaneous blowup of all components is shown for radially symmetric solutions, while a component-wisely different blowup mechanism is suspected by a formal asymptotic analysis. We take this system as a model of tumor microenvironment associated with macrophage at the stage of intravasation. Thus we study the general system of chemotaxis,

$$\begin{aligned} &\partial_t u_i = d_i \Delta u_i - \chi_i \nabla \cdot u_i \nabla v \quad && \text{in } \Omega \times (0,T) \\ &d_i \frac{\partial u_i}{\partial \nu} - \chi_i u_i \frac{\partial v}{\partial \nu} = 0 && \text{on } \partial\Omega \times (0,T) \\ &u_i|_{t=0} = u_{i0}(x) \geq 0 && \text{in } \Omega, \end{aligned} \tag{1}$$

coupled with the Poisson equation

$$-\Delta v = u - \frac{1}{|\Omega|} \int_\Omega u, \quad \left. \frac{\partial v}{\partial \nu} \right|_{\partial\Omega} = 0, \quad \int_\Omega v = 0, \quad u = \sum_{i=1}^N u_i. \tag{2}$$

For this system, we have local-in-time well-posedness and if $T = T_{\max} < +\infty$ is the case, it holds that $\lim_{t \uparrow T} \|u(\cdot,t)\|_\infty = +\infty$, where $\Omega \subset \mathbf{R}^2$ is a bounded domain with smooth boundary $\partial\Omega$, d_i, χ_i, $i = 1, 2, \cdots, N$ are positive constants, and ν is the unit normal vector. Then we shall show the formation of collapse, total mass quantization, subcollapse formation, simultaneous blowup, and particularly, collapse mass separation for radially symmetric case; joint work with Angela Stevens.