Spectrum comparison for a reaction-diffusion system with conservation of a mass

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ABSTRACT

We consider the following two-component system of reaction-diffusion equations

$$\begin{cases} u_t = d_1 \Delta u - g_1(u,v) + g_2(u,v), \\ v_t = d_2 \Delta v + g_1(u,v) - g_2(u,v), \end{cases}$$

in a bounded domain Ω with the homogeneous Neumann boundary conditions. This system has conservation of a mass, namely, the integral of $u + v$ over the domain is preserved for t. With an appropriate choice of g_1, g_2 this system allows a Turing-type instability, that is, the diffusion induces the instability of a constant steady state, and then a localized pattern emerges. In this lecture we first review a background of this model system. We second state basic properties of the system briefly. Then we discuss the mechanism of the pattern formation. In particular, if $g_1 = f(u), g_2 = v$, the Morse index of the steady state solution of the system coincides with the one of an associate scalar reaction-diffusion equation with a nonlocal term. More precisely, we compare eigenvalues of the both linearized problems and show the correspondence of the number of unstable eigenvalues. Studying the associate scalar equation with the nonlocal term, we prove that the system allows a stable nonconstant solution. We also consider another case of nonlinear terms, in which a time periodic motion is observed numerically for appropriate parameter values. Some of the main results is due to the joint work with Professor Toshiyuki Ogawa.