Global existence and boundedness of solutions to chemotaxis systems with general sensitivity

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This talk is based on the joint work with Prof. Kentarou Fujie.

In this talk, we consider solutions to the following chemotaxis system with general sensitivity.

$$\begin{aligned}&\tau u_t = \Delta u - \nabla \cdot (u \nabla \chi(v)) \quad \text{in } \Omega \times (0,\infty),\\ &\eta v_t = \Delta v - v + u \quad \text{in } \Omega \times (0,\infty)\end{aligned}$$

with the Neumann boundary condition. Here, τ and η are non-negative constants, χ is a smooth function on $(0,\infty)$ satisfying $\chi'(\cdot) > 0$ and Ω is a bounded domain in $\mathbf{R}^n$ $(n \geq 2)$.

It is well known that the chemotaxis system with direct sensitivity ($\chi(v) = \chi_0 v$, $\chi_0 > 0$) has blowup solutions in the case where $n \geq 2$. On the other hand, there are many results of research on sufficient conditions for the system to have no blowup solutions. For example, in the case where $n = 2$, $\chi(v) = \chi_0 \log v$ with $0 < \chi_0 \ll 1$ and $\frac{1}{(1+v)^k}$ with $k > 1$, any solution to the system exists globally in time and is bounded.

From these research, people expect that in two dimensional case the systems have no blowup solutions, if $\lim_{v\to\infty} \chi'(v) = 0$. We will talk some results on this conjecture.