# Four Dimensional Topology 

September 6 - September 10, 2018<br>Room E408, Department of Mathematics, Osaka City University

Abstracts<br>Scott Carter (University of South Alabama)<br>The language of surfaces

A system of fonts that describes properly embedded surfaces in 3 -space will be introduced via categorical considerations. The iconography considers dots along a line as 1-morphisms, Temperley-Lieb like diagrams as 2-morphisms, and transformations between these as 3 -morphisms. The transformations will be described via icons that depict a simple closed curve that is being created or annihilated, two types of saddle transformations that occur, four possible types of cusps, and exchanges of critical events. As the iconography is developed, it will be easy to describe the 4 -isomorphisms that generate isotopy.

We will prove:
The naturally monoidal, strictly 2 -pivotal, weakly 3 -pivotal, rotationally commutative, strictly 3 -tortile 3 -category with one object that is freely generated by a weakly self-invertible non-identity 1 -morphism is equivalent to the 3 -category of isotopy classes of properly embedded surfaces in the 3-dimensional space that is the product of the plane with an interval.

## Dale Koenig (Okinawa Institute of Science and Technology) <br> 3-manifolds in trisected 4-manifolds

The spine of a trisected 4-manifold is a singular 3-dimensional set from which the trisection itself can be reconstructed. 3-manifolds embedded in the trisected 4-manifold can often be isotoped to lie almost or entirely in the spine of the trisection. I will define this notion and show that in fact every 3 manifold can be embedded to lie almost in the spine of a minimal genus trisection of some connect sum of $S^{2} \tilde{\times} S^{2}$ s. This mirrors the known fact that every 3 -manifold can be smoothly embedded in a connect sum of $S^{2} \times S^{2}$ s. My methods give an upper bound for how many copies of $S^{2} \tilde{\times} S^{2}$ based on a distance calculated in an appropriately defined graph. I will give several examples and analyze more closely the case of lens spaces lying almost in the spine of trisected 4-manifolds.

## Yuichi Yamada (The University of Electro-Communications) <br> Exceptional Dehn surgeries along certain two-component links related to 4-manifolds

We study Dehn surgeries along some two component links related to the theory of 4-manifolds, and make a complete list of exceptional, i.e., non-hyperbolic integral Dehn surgeries along them. We are interested in the distribution of lens space, Seifert and
graph manifold surgeries. We use Martelli-Petronio-Roukema's theorem on exceptional Dehn surgeries along the minimally twisted four chain link.

Tetsuya Abe (Ritsumeikan University)
How to calculate the enhancement to the Milnor number for fibered links (joint work with Keiji Tagami (Tokyo University of Science))

In 1987, Rudolph introduced the enhancement to the Milnor number for fibered links. In this short talk, I will explain how to calculate the enhancement to the Milnor number for fibered links from the monodromy of a given fibered link.

Tetsuya Ito (Kyoto University)
Strongly quasipositive and quasipositive links and the defect of Bennequin's inequality
(joint work with Keiko Kawamuro (Univ. Iowa) and Jesse Hamer(Univ. Iowa))
We discuss several conjectural implications among various positivities of knots and links, such as strongly quasipositivity or quasipositivity. These positivities are related to the defect of (slice) Bennequin inequality. We give several supporting evidences for our conjectural implications, present several techniques to find a strongly quasipositive or a quasipositive braid representative of knots and links.

## Keiko Kawamuro (University of Iowa) <br> The fractional Dehn twist coefficients and branched coverings <br> (joint work with Tetsuya Ito (Kyoto University) )

I discuss how the fractional Dehn twist coefficient behaves under a fully ramified branched covering of an open book, and give applications to both topological and contact manifolds. Among them, we show that non-right-veering closed braids represent virtually loose transverse links.

## Takahiro Oba (Kyoto University)

## Surfaces in $D^{4}$ with the same boundary and fundamental group

This talk is concerned with symplectic surfaces in a symplectic 4 -disk ( $D^{4}, ¥$ omega) bounded by the same transverse link in the standard contact 3 -sphere $\left(S^{3}, \xi_{s t}\right)$. There are some examples of transverse links (or knots) bounding more than one symplectic surface up to isotopy. All these surfaces can be distinguished by the fundamental groups of their complements. In this talk, I will present a family of pairs of distinct two symplectic surfaces whose boundaries are the same transverse knot and whose complements have isomorphic fundamental groups. To distinguish the two surfaces of each pair, I take double branched covers branched along them.

## Osamu Saeki (Kyushu University)

Simplifying broken Lefschetz fibrations and trisections of 4-manifolds
(joint work with R. İnanç Baykur (University of Massachusetts))

We present explicit algorithms for simplifying the topology of indefinite fibrations on smooth 4-manifolds, which include broken Lefschetz fibrations and indefinite generic maps, from the viewpoint of singularity theory. These algorithms allow us to give purely topological and constructive proofs of the existence of simplified broken Lefschetz fibrations on general 4-manifolds, and a theorem of Auroux-Donaldson-Katzarkov on the existence of simplified broken Lefschetz pencils on near-symplectic 4-manifolds. We moreover establish a correspondence between broken Lefschetz fibrations and trisections of 4 -manifolds, and show the existence of simplified trisections on all 4-manifolds.

## Selman Akbulut (Michigan State University) A simple family of infinitely many absolutely exotic manifolds

I will demonstrate a smooth 4 -manifold $M$, obtained by attaching a 2 -handle to $B^{4}$ along a certain knot $K \subset \partial B^{4}$, which admits infinitely many absolutely exotic copies $M_{n}, n=0,1,2 .$. , such that each copy $M_{n}$ is obtained by attaching 2 -handle to a fixed compact smooth contractible manifold $W$ along the iterates $f^{n}(c)$ of a knot $c \subset W$ by a diffeomorphism $f: \partial W \rightarrow \partial W$. This generalizes the example I gave in "An exotic 4 -manifold, Journ. of Diff. Geom. 33, (1991)" which corresponds to the $n=1$ case.

## Shin Satoh (Kobe University) <br> The $n$-cable of a ribbon 2-knot

We define the $n$-cable $K(n)$ associated with a 2 -knot $K$ and an integer $n \geq 2$. It is a kind of a satellite 2 -knot with the companion $K$. If $K$ is a ribbon 2 -knot, then so is $K(n)$. We will study how to get a welded arc presentation of $K(n)$ from that of $K$.

Jieon Kim (Pusan National University)<br>Biquasile colorings of oriented surface-links<br>(joint work with Sam Nelson (Claremont McKenna College))

A biquasile is a set with six binary operations satisfying the conditions derived from Reidemeister moves. In this talk, we introduce a biquasile coloring for marked graph diagrams of oriented surface-links and counting invariants of oriented surface-links. We use these colorings to define counting invariants and Boltzmann enhancements of the biquasile counting invariants for oriented surface-links.

Sang Youl Lee (Pusan National University)

## On invariants for surface-links via quandle cocycle invariants for classical links

A marked graph diagram is a link diagram possibly with some 4 -valent vertices equipped with markers. It is known that a marked graph diagram presents a surfacelink in $\mathbb{R}^{4}$ modulo Yoshikawa moves. In this talk, we introduce a polynomial $\ll D \gg$ for a marked graph diagram $D$ defined by a state-sum model with a given quandle cocycle invariant for knots and links in $\mathbb{R}^{3}$ as its state evaluation, and then give an invariant for surface-links in $\mathbb{R}^{4}$ derived from the polynomial $\ll D \gg$.

Ki-Heon Yun (Sungshin Women's University)

## On the minimal number of singular fibers in Lefschetz fibrations over the torus

In the talk we will show that the minimal number of singular fibers $N(g, 1)$ in a genus- $g$ Lefschetz fibration over the torus is at least 3 . As an application, we show that $N(g, 1) \in\{3,4\}$ for $g \geq 5, N(g, 1) \in\{3,4,5\}$ for $g=3,4$ and $N(2,1)=7$. This is a joint work with Prof. Stipsicz.

## Hironobu Naoe (Tohoku University)

## Closed 4-manifolds with shadow-complexity one

(joint work with Yuya Koda (Hiroshima University) and Bruno Martelli (The University of Pisa))

Any closed oriented smooth 4-manifold is represented by a 2-dimensional polyhedron called a shadow, using which, the shadow-complexity of a 4 -manifold is defined. It measures how complicated the 2 -skelton of the 4 -manifold is. We characterize all the closed 4-manifold having shadow-complexity $\leq 1$.

## Eylem Zeliha Yildiz (Michigan State University) <br> Knot concordance in 3-manifolds

I will discuss PL and smooth knot concordances in 3-manifolds. In particular I will show that all the knots in the free homotopy class of $S^{1} \times p t$ in $S^{1} \times S^{2}$ are concordant to each other. I will also discuss an application of this concordances to constructing exotic 4-manifolds.

## Akiko Shima (Tokai University) The structure of a minimal $n$-chart with two crossings <br> (joint work with Teruo Nagase (Tokai University))

We investigate the structure of minimal charts with two crossings. Let $\Gamma$ be a chart and $m$ a label of $\Gamma$. We define $\Gamma_{m}$ by the union of all the edges of label $m$ and their vertices.

Let $\Gamma$ be a minimal $n$-chart with exactly two crossings in a disk $D^{2}$. Then we can show that by applying C-I-M1 moves and C-I-M2 moves, we can find an annulus $A$ containing all the white vertices of $\Gamma$ but not intersecting any hoops nor free edges such that (1) each connected component of $C l\left(D^{2}-A\right)$ contains a crossing, (2) $\Gamma \cap \partial A$ consists of eight points.

Moreover we can show the annulus $A$ can be split into mutually disjoint disks $D_{1}, D_{2}, D_{3}, D_{4}$ and mutually disjoint disks $E_{1}, E_{2}, E_{3}, E_{4}$ such that
(3) there exists two labels $1 \leq \alpha<\beta \leq n-1$ such that for each $i=1,2,3,4$, $\Gamma \cap E_{i}=\emptyset$ or $\Gamma \cap E_{i}$ consists of parallel proper arcs of $E_{i}$ of label $\alpha+1$, or the tangle $\left(\Gamma \cap E_{i}, E_{i}\right)$ is a net-tangle with $\Gamma \cap E_{i} \subset \cup_{j=\alpha+1}^{\beta-1} \Gamma_{j}$,
(4) for $i=1,3$, the tangle $\left(\Gamma \cap D_{i}, D_{i}\right)$ is an IO-tangle of label $\alpha$ with $\Gamma \cap D_{i} \subset$ $\Gamma_{\alpha} \cup \Gamma_{\alpha+1}$ and $\Gamma_{\alpha} \cap \partial D_{i}=$ two points,
(5) for $i=2,4$, the tangle $\left(\Gamma \cap D_{i}, D_{i}\right)$ is an IO-tangle of label $\beta$ with $\Gamma \cap D_{i} \subset$ $\Gamma_{\beta} \cup \Gamma_{\beta-1}$ and $\Gamma_{\beta} \cap \partial D_{i}=$ two points.

As important results, we can calculate the fundamental group of the exterior of the surface link represented by $\Gamma$, and the braid monodromy of the surface braid represented by $\Gamma$.

## Sukuse Abe (Osaka City University)

## Relations between quandle shadow cocycle invariants and vassiliev invariants

We conduce vassiliev invariants by quandle shadow cocycle invariants by using Alexander quandle of classical links. Generally, finite type invariants (Vassiliev invariants) of classical knots are not compatible with quandle cocycle invariants well. When, for the knots as closure (or plat closure) of braids that are composed of $m$ strings, we make the number of the crossings larger, the values of quandle cocycle invariants are bounded, while the values of Vassiliev invariants increase by the order of the polynomial functions of the crossing numbers. However, vassliev invariants deduced from shadow cocycle invariants can be defined independently of the way to choose the ideals of Alexander quandles. Therefore, the values of these Vassliev invariants are not trivial because the same number of them as the (infinite) ideals is deduced from 1 shadow cocycle invariant. Then, we will try to apply them to surface-knots. We can calculate Vassliev invariants of twist spun knots. Our questions are as follows: (1) Do singular curves corresponding to singular points exist? (2) For 2-knots, can quantum invariants be defined?

Mizuki Fukuda (Tohoku University) On the Gluck twist along branched twist spins

A Montesinos twin is a pair of 2-knots which meet transversely twice in the 4 -sphere. A branched twist spin is an example of a part of such a twin. In the study of four dimensional topology, the Gluck twist which is a surgery along a 2 -knot in the 4 -sphere is well-studied. In this talk, we determine the 2-knot which obtained from a branched twist spin by the Gluck twist.

## Riccardo Piergallini (Università di Camerino) <br> Four-manifolds as branched covers

The talk will concern some old and new results about representing four-manifolds in terms of branched coverings. In particular, it will be discussed how the most known representation theorem of closed smooth four-manifold as covers of the four-sphere can be extended to the cases of bounded, open, and topological four-manifolds, as well as to branched covers of different basic four-manifolds in the closed smooth case.

## Ash Lightfoot (NRU Higher School of Economics, Moscow) <br> Link homotopy in 4 -space via chart descriptions

S. Kamada introduced chart diagrams to describe two-dimensional braids, which (roughly speaking) are to classical braids what Cerf diagrams are to Morse functions. In this talk we recall chart diagrams, and discuss their application in defining Vassiliev invariants and approaching problems in link homotopy of 2 -spheres in the 4 -sphere.

## Mark Hughes (Brigham Young University)

## Braided surfaces with caps and positive branch points

In this talk we will define braided surfaces with caps, which generalizes the notion of braided surfaces to nonribbon surfaces in $D^{4}$. We show that any surface in $D^{4}$ can be isotoped to a braided surface with caps with only positive branch points, and that this isotopy can be taken rel boundary if the boundary is already a classical closed braid in $S^{3}$. We will then discuss some applications of these surfaces to constructing Lefschetz fibrations with prescribed boundary open book decompositions and finding (near) symplectic surfaces in $C P^{2}$.

## Seiichi Kamada (Osaka City University) On surfaces immersed in 4-space

An immersed surface-link is a generically immersed closed surface in Euclidian 4space. We introduce a lemma on isotopic deformations of immersed surface-links. As an application, we obtain Markov type theorem on braid presentations and Reidemeister type theorem on diagrams for immersed surface-links. We also introduce multiplication of immersed surface-links.

## Hokuto Konno (The University of Tokyo)

## A family version of the Bauer-Furuta invariant and the complex of surfaces

In 2017 I have defined an abstract simplicial complex for a given $\operatorname{spin}^{c} 4$-manifold by considering embedded surfaces violating the adjunction inequalities. On the simplicial complex, I also defined a cohomology class, which is regarded as a family version of the Seiberg-Witten invariant. In this talk I will explain we can also define a family version of the Bauer-Furuta invariant on this simplicial complex. This is a joint work with Tirasan Khandhawit.

## Kouichi Yasui (Osaka University) <br> Geometrically simply connected 4 -manifolds and stable cohomotopy Seiberg-Witten invariants

A geometrically simply connected 4 -manifold is a 4 -manifold admitting a handle decomposition without 1-handles, and there are no known simply connected closed 4manifolds that are not geometrically simply connected. In this talk, we show that every geometrically simply connected positive definite closed 4 -manifold with $b_{2}^{+}>1$ has a vanishing stable cohomotopy Seiberg-Witten invariant, and thus admits no symplectic structure. We also show that every geometrically simply connected closed oriented 4manifold with $b_{2}^{+} \not \equiv 1$ and $b_{2}^{-} \not \equiv 1(\bmod 4)$ admits no symplectic structure for at least one orientation of the manifold.

## Motoo Tange (University of Tsukuba) <br> On ribbon disks in handle decompositions

We put a holed ribbon disk in a 4-dimensional a handle decomposition. In this talk, we describe how the disk behaves by the handle moves and isotopy.

## Kenjiro Sasaki (Kyoto University)

## Canonical representations and the universal families of Riemann surfaces

The universal family over a moduli space of Riemann surfaces is locally described by using Takamura's linearization: The universal family around a Riemann surface is linearly approximated by a fibration (with singular fibers) constructed from the representation of the automorphism group of the Riemann surface on the space of holomorphic quadratic differentials ("canonical representation"). We show that the canonical representations for the Riemann surfaces with "particular" automorphism groups (sporadic simple groups, symmetric groups, etc.) are reducible. The universal families around these Riemann surfaces are decomposed into simple pieces.

## Shigeru Takamura (Kyoto University) Quotient families and their applications

We explain recent developments of the theory of quotient families, including "visual description" of the universal families over the moduli spaces of Riemann surfaces. Linearization and stratification of quotient families are fundamental tools to describe them. In the local descriptions of quotient families, "linear approximation" via linear quotient families plays the key role; they are constructed from finite group actions together with linear representations of the finite groups. Quotient families admit natural stratifications, via which "specializations of various objects" (singularities, singular fibers and subfamilies) are defined and described. These specializations are deeply related to the structures of the subgroup poset of stabilizers.

## Naoki Kitazawa (Kyushu University) <br> 4-manifolds admitting special generic maps into the 3 -dimensional Euclidean space

A special generic map is a smooth map regarded as a natural generalization of Morse functions with just 2 singular points on homotopy spheres: canonical projections of unit spheres are simplest examples.

As a simple case, manifolds admitting such maps into the plane are completely classified by Saeki in 1993; they are represented as connected sums of manifolds represented as the total spaces of bundles over the circle whose fibers are homotopy spheres and the images are regarded as orientable compact surfaces.

Moreover, as a general property, diffeomorphism types of manifolds including homotopy spheres admitting special generic maps are often restricted in the case where the dimensions of the target spaces are high enough and this makes special generic maps attractive objects both in the global singularity theory of smooth maps and application to differential topology of manifolds.

Motivated by such studies and results on special generic maps, we study about general properties of special generic maps into the 3 -dimensional spaces, where there are several studies for simple explicit cases by Saeki and Sakuma etc.. As a main result, we show two equivalent conditions for 4 -dimensional manifolds to admit special generic maps into the 3 -dimensional Euclidean space by appropriate smooth or PL maps into or onto 2 -dimensional spaces. It is a theorem of a new type on a characterization of manifolds admitting special generic maps.

We also present the meanings of this new theorem both in differential topology of (4-)manifolds and the global singularity theory of smooth maps.

Last, this talk is based on the following preprint;
N. Kitazawa, A new explicit way of obtaining special generic maps into the 3dimensional Euclidean space, arxiv:1806.04581v3.

Kengo Kawamura (Osaka City University)
A generating set of oriented Roseman moves
Roseman moves are seven types of local modifications for surface-link diagrams in 3 -space which generate ambient isotopies for surface-links in 4 -space. They are generalization of three types of Reidemeister moves for link diagrams in 2-plane which generate ambient isotopies for links in 3 -space. A (minimal) generating set of oriented Reidemeister moves is introduced by Polyak. In this talk, we introduce a generating set of oriented Roseman moves.

## Inasa Nakamura (Kanazawa University)

## Simplifying numbers of branched covering surface-knots

A branched covering surface-knot is a surface-knot in the form of a branched covering over an oriented surface-knot. We can simplify a branched covering surface-knot by an addition of 1-handles with chart loops to a form such that its chart is the union of edges whose end points are vertices of degree one, and 1-handles with chart loops. The simplifying number is the minimum number of 1 -handles necessary to obtain such a simplified form. In this talk, we give upper estimates of simplifying numbers.

Kent Orr (Indiana University)

## New Perspectives on an Old Problem of Milnor <br> (joint work with Jae Choon Cha (Postech University))

In the mid-1950s, Milnor introduced his link invariants, a vast and profound extension of the classical linking number. His examples, results and a seminal list of problems have driven decades of research. One key question Milnor posed has remained open. How can one extract a version of Milnor invariants from the transfinite lower central series? We consider this problem anew and present a solution in a broad setting. We include detailed computations for key and illustrative examples, as well as, time allotting, a brief outline of past progress on Milnor's invariants.

Jongil Park (Seoul National University)
On symplectic fillings of a quotient surface singularity

One of active research areas in symplectic 4 -manifolds is to cassify symplectic fillings of certain 3 -manifolds equipped with a contact structure. Among them, people have long studied symplectic fillings of the link of a quotient surface singularity. Note that the link of a quotient surface singularity carries a canonical contact structure which is also known as the Milnor fillable contact structure. One the other hand, algebraic geometers also have studied Milnor fibers as a general fiber of smoothings for a quotient surface singularity.

In this talk, I'd like to review known results on symplectic fillings of quotient surface singularities and to explain a relation between minimal symplectic fillings and the Milnor fibers of quotient surface singularities. This is a joint work with Heesang Park, Dongsoo Shin, and Giancarlo Urzúa.

Note. The following talk was canceled due to a big typhoon.

## Sudipta Kolay (Georgia Tech) Braided Embeddings

We will introduce the notion of braided embeddings of manifolds, which generalizes the notion of a closed braid. We will talk about isotopy problem for braided embeddings, and and discuss braid index of surface knots.

Organizers: Seiichi Kamada, Kouichi Yasui, Takao Matumoto and Kengo Kawamura
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