Results of my research

Hiromasa Moriuchi

A link is n circles $S^1 \cup \cdots \cup S^1$ embedded in the 3-sphere S^3. As a generalization of classical knot theory, we can consider other objects embed in S^3. A spatial graph is a graph embedded in S^3. If the graph consists of two vertices and three edges such that each edge joins the vertices, then it is called a θ-curve. Moreover, if the graph consists of two loops and an edge jointing the vertices of each loop, then it is called a handcuff graph.

In knot theory, there exists the study of creating a prime knot table. Similarly, there exists the study of making a table of spatial graphs. In particular, some researchers worked on making a θ-curve table. J. Simon made a table of θ-curves with up to five crossings in 1987, R. A. Litherland announced a table of prime θ-curves with up to seven crossings in his letter of 1989, T. Harikae enumerated special θ-curves with up to nine crossings in 1987. However, we can suppose that their θ-curves obtained by adding an edge to some knot. Moreover, there was no published proof of the completeness of Litherland's table. Then I considered to complete the table of prime θ-curves with up to seven crossings.

I applied Conway's method to enumerate θ-curves. In 1969, J. H. Conway made an enumeration of prime links by introducing the concept of a tangle and a basic polyhedron. Here, a tangle is a pair (B^3, t), where t is a 1-manifold properly embedded in a unit 3-ball B^3 with boundary components. A basic polyhedron is a connected 4-regular graph embedded in a 2-sphere, which has no bigon. We can obtain links from basic polyhedra by substituting tangles for their vertices. By using Conway's method, we can enumerate all links in order of crossing numbers thoroughly.

In order to apply Conway's method, we need the following works. First, I enumerated algebraic tangles with up to seven crossings (see [4]). Second, I constructed a prime basic θ-polyhedron to enumerate prime θ-curves. Here, a θ-polyhedron is a connected planar graph embedded in a 2-sphere, whose two vertices are 3-valent, and the rest are 4-valent. Then our θ-polyhedron is different from Conway's polyhedron. Since I considered to produce a prime θ-curve table, I omitted non-prime θ-polyhedra. Then there exist twenty-four prime basic θ-polyhedra with up to seven 4-valent vertices. We can obtain θ-curves from prime basic θ-polyhedra by substituting tangles for their 4-valent vertices. I obtained all the prime θ-curves with up to seven crossings, which are the same table as Litherland's. Litherland classified these θ-curves by constituent knots and the Alexander polynomial. I classified them by the Yamada polynomial (see [5]). Moreover, I first enumerated all the prime handcuff graphs with up to seven crossings in similar way. I also classified these handcuff graphs by using the Yamada polynomial (see [3]).

For any two given 3-valent vertices, we need to consider at most six different order-3 vertex connected sums. I enumerated non-prime θ-curves and handcuff graphs with up to seven crossings by using non-prime *theta*-polyhedra. However, we can not classify all of them by calculating the Alexander polynomial and the Yamada polynomial because of the influence on vertex connected sums. I classified non-prime θ-curves and handcuff graphs with up to six crossings by observing their constituent links. From this work, A. Ishii, K. Kishimoto, M. Suzuki, and I made a table of "handlebody-knot" with up to six crossings (see [7]).

Kinoshita's theta-curve $\theta(1,1,1)$ is well known as locally unknotted theta-curve. We add full-twists to $\theta(1,1,1)$, and obtain generalized Kinoshita's theta-curve $\theta(i,j,k)$. A generalized Kinoshita's theta-curve $\theta(i,j,k)$ has some symmetries. I classified $\theta(i,j,k)$ by using the multiset of polynomials which is related to the Kojima-Yamasaki η-function. Moreover, I classified the order-3 vertex connected sum of two generalized Kinoshita's theta-curves $\theta(i,j,k)$ and $\theta(i',j',k')$ by the multiset.