Research Results

I am working on discretization of surfaces based on integrable systems techniques. In particular, I am interested in discretization of linear Weingarten surfaces. So far I have shown several results as follows (references can be found in "List of Papers"):

1. Research on semi-discrete minimal surfaces in Euclidean 3-space ([1])

Müller, Wallner introduced semi-discrete isothermic surfaces in Euclidean 3-space and defined semi-discrete minimal surfaces. In a joint work with Wayne Rossman (Kobe University), we gave a Weierstrass representation for semi-discrete minimal surfaces and constructed new semi-discrete minimal surfaces. As an application, we discretized catenoid via various methods. In particular, we showed that semi-discrete minimal catenoids in the sense of Müller, Wallner have the same profile curves as both discrete and smooth catenoids. This indicates that semi-discrete surfaces help us to recognize the similarities and differences between smooth and discrete surfaces.

2. Research on discretized maximal surfaces in Minkowski 3-space ([2], [8])

We discretized (and semi-discretized) maximal surfaces (spacelike surfaces with mean curvature identically 0) as a special class in isothermic surfaces in Minkowski 3-space. Unlike the case of smooth minimal surfaces in Euclidean 3-space, maximal surfaces generally have singularities, so it is natural to expect that discrete (or semi-discrete) maximal surfaces have singularities. We gave Weierstrass representations for discrete and semi-discrete maximal surfaces, and described their singularities. Singularities of discretized surfaces had not been explicitly defined before this. Based on these results, we aim to describe singularities of general discretized surfaces.

3. Research on discrete linear Weingarten surfaces in 3-dimensional spaceforms ([5])

With Wayne Rossman, we analyzed the discrete linear Weingarten surfaces of Bryant type in hyperbolic 3-space introduced by Hoffmann, Rossman, Sasaki and Yoshida. Linear Weingarten surfaces of Bryant type are linear Weingarten surfaces which lie in a particular deformation family between flat surfaces and discrete constant mean curvature (CMC) 1 surfaces. We considered singularities of discrete linear Weingarten surfaces of Bryant type and their unit normal vector fields in de Sitter 3-space. In particular, in a certain situation, we showed that the condition for singularities of discrete CMC 1 surfaces in de Sitter 3-space to appear is the same as the one for discrete maximal surfaces.

4. Research on discrete CMC surfaces in 3-dimensional Riemannian spaceforms ([9])

Hoffmann introduced a construction of discrete non-zero constant mean curvature surfaces in Euclidean 3-space using matrix factorizations. However, there are several gaps in the proofs, so it is not clear that any discrete constant mean curvature surface can be constructed via his method. With Yuta Ogata (Kobe University) [9], we completely filled the gaps and, except for special cases, we extended the construction to discrete constant mean curvature surfaces in 3-dimensional Riemannian spaceforms. This construction gives a geometric solution of a discrete version of a sinh-Gordon equation proposed by Pedit, Wu. Moreover, we analyzed singularities of discrete constant positive Gaussian curvature surfaces obtained by parallel surfaces of discrete constant mean curvature surfaces. This result is highly related to the analysis of discrete sinh-Gordon equation.

In addition, we classified semi-discrete surfaces of revolution in Euclidean and Minkowski 3-spaces, derived Weierstrass-type representations for smooth timelike constant mean curvature surfaces in Minkowski and AdS 3-spaces and analyzed their singularities, and proposed a new theory of discrete timelike surfaces ([7]).