Plans

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(1) Alexander matrices and the Alexander polynomials of 2-knots

Using the Wirtinger presentation as a presentation of the fundamental group for the complement of a knot, the Alexander matrix becomes an $(n-1) \times n$ matrix, and the determinant of an $(n-1) \times (n-1)$ submatrix obtained by removing a column becomes the Alexander polynomial. Recently, Ishii and Oshiro have formulated such an operation for matrices. They construct a theory to make a square matrix from an arbitrary matrix with its relations between rows or/and columns (A. Ishii and K. Oshiro, *Augmented Alexander matrices and generalizations of twisted Alexander invariants and quandle cocycle invariants*, preprint.). By this theory, since we can take the "determinant" from a general matrix such as an Alexander matrix of a 2-knot, we expect to be able to introduce invariants of 2-knots. Therefore, I attempt to research the Alexander polynomial of a 2-knot based on this theory, a joint work with Oshiro.

(2) Generalization of equivalence for certain 2-knots

In 2-dimensional knot theory, it is important to determine whether given two 2-knots are equivalent or not. We may show that two 2-knots are not equivalent if we choose an invariant of a 2-knot well. On the other hand, it is sufficient to verified that two 2-knots are indeed related by equivalent deformations in order to show that they are equivalent. However, it is not easy to deform into each other even if one knows that they are equivalent.

Up to now, several methods to construct a 2-knot from a 1-knot have been developed, and then it is known that there exist equivalent 2-knots obtained from different constructions. For example, it is shown that the 3-twist-spun trefoil knot $\tau^3(3_1)$ is equivalent to the half-roll-spun figure-eight knot $\rho^{\frac{1}{2}}(4_1)$ (Fox's rolling). The point of the proof is to use an analogy and symmetry of a trefoil knot and a figure-eight knot. Thus, we generalize it and try to construct similar examples.

(3) Bridge trisections for surface-knots

A trisection of a 4-manifold is to decompose it into three 4-dimensional handlebodies, which is generalization of a Heegaard splitting of a 3-manifold. For example, the 4-sphere S^4 can be decomposed into three 4-balls B^4. Using this trisection, every surface-knot in S^4 can be decomposed into three trivial disk systems, each of which is contained in B^4. We call such a spliting a bridge trisection of a surface-knot, and then we define an invariant of a surface-knot called the bride number.

A bridge trisection of a surface-knot is introduced by Meier and Zupan (Bridge trisections of knotted surfaces in S^4. Trans. Amer. Math. Soc., 2017). They constructed bridge trisections of a spun 2-knot and a twist-spun 2-knot and investigated their bridge numbers. Moreover, they proved that there exist infinitely many distinct 2-knots with arbitrary large bridge number. By their work, we expect to be able to obtain similar results for other concrete 2-knots. Hence, I attempt to study bridge trisections and the bridge numbers of a ribbon 2-knot and a roll-spun 2-knot. Furthermore, I try to develop how to calculate some invariants of surface-knots from bridge trisections.