Research Results

Hessenberg varieties are subvarieties (singular in general) of a flag variety for any Lie types, which are relatively new research subject. Their topology make connections with other research areas. In what follows, I state a part of the research results.

(1) The cohomorogy ring of Peterson variety in all Lie types
(joint work with Megumi Harada and Mikiya Masuda)
It is known that the Peterson variety is related with the quantum cohomology of flag varieties. In type A, an explicit presentation by generators and fundamental relations of the cohomology rings of the Peterson variety was given by Fukukawa-Harada-Masuda. We gave an explicit presentation of the cohomology rings of the Peterson variety in all Lie types which generalizes the presentation by Fukukawa-Harada-Masuda.

(2) The cohomorogy ring of regular nilpotent Hessenberg varieties in type A
(joint work with Hiraku Abe, Megumi Harada, and Mikiya Masuda)
The regular nilpotent Hessenberg varieties are the family of the flag variety connecting between the Peterson variety (mentioned in (1)) and the flag variety itself. We gave an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in type A.

(3) The regular nilpotent Hessenberg varieties and regular semisimple Hessenberg varieties in type A
(joint work with Hiraku Abe, Megumi Harada, and Mikiya Masuda)
The cohomology ring of regular semisimple Hessenberg varieties admits an action of the Weyl group via GKM technique, in particular, when the case of type A, it is known that this action has a beautiful connection with the Stanley's chromatic symmetric function in graph theory. Using an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in type A given in (2), we obtained that in type A the cohomology rings of regular nilpotent Hessenberg varieties are isomorphic to the invariant subring of the cohomology rings of regular semisimple Hessenberg varieties under the symmetric group action.

(4) Hessenberg varieties and hyperplane arrangements
(joint work with Takuro Abe, Mikiya Masuda, Satoshi Murai, and Takashi Sato)
We obtained the surprising result that in all Lie types the cohomology rings of regular nilpotent Hessenberg varieties can be described in terms of hyperplane arrangements. From this result, we proved that the ring isomorphism explained in (3) in all Lie types, Peterson's announcement and Sommers-Tymoczko conjecture. Moreover, this surprising connection represents a significant step towards the solution for the problem of giving explicit presentations of the cohomology rings of regular nilpotent Hessenberg varieties. In fact, we obtained presentations for types B, C, G.

(5) The cohomorogy ring of regular nilpotent Hessenberg varieties in all Lie types
(joint work with Makoto Enokizono, Takahiro Nagaoka, and Akiyoshi Tsuchiya)
As explained in (4), from the point of view of hyperplane arrangements we gave an explicit presentation of the cohomology rings of regular nilpotent Hessenberg varieties in all Lie types.