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1 Previous research

The Cauchy problem of Nonlinear differential equations may not admit global solutions in general, as can be seen by a simple counterexample $v(t) = \{1/v(0) - t\}^{-1}$, which solves an ODE: $v' = v^2$, $t > 0$, and is defined only for $0 < t < T := 1/v(0)$. A singularity formation appears in various nonlinear PDEs. Since it is a consequence of the fact that some key quantities become infinity in a finite time, we call it **blow-up**. Fundamental problems in understanding blow-up phenomena are:

- whether or not a blow-up actually occurs
- investigating the space-time positions at which the blow-up takes place
- describing the asymptotic behavior of blow-up solutions.

The last problem tends to require, in general, delicate analysis and hence there are many open problems on that topic. Various nonlinear PDEs that come from physical applications are endowed with some symmetry properties and are invariant under **scaling transformation** (i.e., the action of the multiplicative group of the whole $\mathbf{R}_+$). For instance, if u is a solution of the Fujita equation $u_t = \Delta u + |u|^{p-1}u$, then so is

$$u_\lambda(x,t) := \lambda^{1/(p-1)} u(\lambda x, \lambda^2 t) \qquad (\lambda > 0).$$

A solution u is called of self-similar if it is invariant under the self-similar transformation, that is, $u \equiv u_\lambda$ for all $\lambda > 0$. Self-similar solutions exhibit particular structures of the equation and predict the generic behaviors for some class of solutions. Type I blow-up solutions (self-similar solutions associated to the scaling structure) attracted much attention until 1990s, whereas **type II blow-up solutions** (i.e., solutions with the blow-up mechanisms different from the scaling structure) have been studied since 2000s. It was revealed that such solutions played a central role in understanding nonlinear phenomena such as **chemotactic aggregation** in a biological model. The understanding of type II blow-up is, however, far from well understood. At present, a major goal in this direction is to understand such a type of singularity through typical examples.

Therefore classification of type II blow-up patterns are quite important in understanding the whole structure of general solutions and its relation to the role played by the nonlinear term. This is one of the goal of my study. I have studied singularity formations in several nonlinear parabolic problems, which include the Fujita equation, harmonic map heat flow, Keller–Segel chemotaxis model, and mean curvature flow. Since existence and properties of type II blow-up solutions heavily depend on the nonlinearity and space dimension, the actual analysis is carried out using particular structures in some model equations. **Method of matched asymptotic expansions** has been used in various nonlinear problems in applied mathematics, which goes back to a work of Prandtl(1905). It is so powerful to construct typical examples in a formal but essential manner and to derive quantitative information of the solutions. The rigorous treatment is known to be a hard task in general, even after deriving a number of nontrivial computations on a priori estimates. Thus, studies using this method have been limited so far in some works of the PDE community. I have developed this technique in various nonlinear problems, resulted in solving some major open problems.