Study results

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In pseudo-Riemannian geometry, a pseudo-Riemannian symmetric space is a typical example of homogeneous spaces. Rossmann, Oshima-Sekiguchi investigated the restricted root systems with respect to maximal split abelian subspaces for semisimple pseudo-Riemannian symmetric spaces. Oshima-Sekiguchi also investigated the structure of a semisimple pseudo-Riemannian symmetric space in terms of the Satake diagram associated with the restricted root system.

In [1], we investigate the restricted root systems and the Satake diagrams for irreducible classical semisimple pseudo-Riemannian symmetric spaces. For an irreducible classical semisimple pseudo-Riemannian symmetric space G/H, we give a recipe to determine the restricted root system Δ of G/H with respect to a maximal split abelian subspace, the Satake diagram of G/H associated with Δ and the Satake diagram of G/H associated with the restricted root system with respect to a Cartan subspace. By using this recipe, for all irreducible classical semisimple pseudo-Riemannian symmetric spaces G/H, we obtain the followings:

(1) the Dynkin diagram of the restricted root system Δ of G/H with respect to a maximal split abelian subspace, and the signature of its simple roots,

(2) the Satake diagrams of G/H associated with Δ and the restricted root system with respect to a Cartan subspace, respectively,

(3) the codimension of a hyperbolic (resp. an elliptic) orbit whose dimension is the maximum among the dimensions of all hyperbolic (resp. elliptic) orbits of the isotropy representation of G/H,

where a hyperbolic (resp. an elliptic) orbit is an orbit through a hyperbolic (resp. an elliptic) element.

In [2], we investigate the local orbit types of orbits of the isotropy representations for semisimple pseudo-Riemannian symmetric spaces. Here, the local orbit type of an orbit is the conjugate class of the Lie algebra of the isotropy group at a point of the orbit. We proved that, for a closed subsystem Δ' of the restricted root system of G/H with respect to a maximal split abelian subspace, there exists a semisimple symmetric pair whose restricted root system with respect to a maximal split abelian subspace is isomorphic to Δ'. We give a recipe to determine the local orbit types of hyperbolic orbits and elliptic orbits for irreducible semisimple pseudo-Riemannian symmetric space G/H in terms of this result. By using this recipe we classified the local orbit types of hyperbolic orbits and elliptic orbits of the isotropy representations for some semisimple pseudo-Riemannian symmetric spaces.