Results of my research

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Pre-Bloch invariants Neumann and Yang defined an element of the Bloch group for a finite volume hyperbolic 3-manifold. In [1], I defined the pre-Bloch invariant for compact 3-manifold with boundary whose genus is greater than one. Since finite volume hyperbolic 3-manifolds have toral boundary, this is a generalization of the invariant of Neumann-Yang.

Let M be a compact 3-manifold with boundary. Let ρ be a representation of the fundamental group $\pi_1(M)$ into $\mathrm{PSL}(2,\mathbb{C})$. For ∂M, fix a pants decomposition C of ∂M. Let $\mathfrak{o}$ be an orientation on the simple closed curves of C. We triangulate each pair of pants into two ideal triangles so that deleted vertices go to the boundaries of pants in the directions of $\mathfrak{o}$. We extend this triangulation to a triangulation of M. The developing map of ρ maps the universal covering $\tilde{M}$ of M to $\mathbb{H}^3$. Then each 3-simplex maps to an ideal 3-simplex in $\mathbb{H}^3$ and we can define an element $\beta(M,\rho,C,\mathfrak{o})$ of the pre-Bloch group $\mathcal{P}(\mathbb{C})$. We show that $\beta(M,\rho,C,\mathfrak{o})$ only depends on M, ρ, C, $\mathfrak{o}$, and not on the choice of triangulation of M.

For each boundary curve γ_i of a pair of pants, we define H_i by the square of the eigenvalue of $\rho(\gamma_i)$. We call this parameter *holonomy parameter*. For γ_i, we also define *twist parameter*. The twist parameter measures how the two pair of pants adjacent to γ_i are glued. By using these parameters, I showed the following variation formula of the volume function:

$$d\mathrm{Vol} = -\frac{1}{2}\sum_{k=1}^{3(g-1)}\big(\log|W_k| d\arg(H_k) - \log|H_k| d\arg(W_k)\big).$$

A method for finding ideal points Yoshida introduced a method for finding ideal points from an ideal triangulation of a 3-manifold. His method gives necessary conditions which the valuation associated to an ideal point satisfies. Since the conditions are given by linear equations, we can get candidates of ideal points only by solving linear equations. But, by using this method, we can only find candidates of ideal points. In fact, there are many examples of candidates linear solutions which do not correspond to ideal points. So we have to check that each of the candidates are actually ideal points.

In [2], I give a criterion for a candidate to correspond actually to an ideal point. We also show how we can compute the number of ideal points corresponding to the candidate. In our method, our task is only to compute determinants of some matrices.

Let $K = \Delta(z_1)\cup\cdots\cup\Delta(z_n)$ be an ideal triangulation of M where z_i are the complex parameters of ideal tetrahedrons. For each edge of K, we have a gluing equation $\prod_{\nu=1}^{n} z_\nu^{r'_{i,\nu}}(1-z_\nu)^{r''_{i,\nu}} = \pm 1$. Let $\mathcal{D}(M,K)$ be an affine algebraic variety defined by the system of equations. For $p \in \mathcal{D}(M,K)$, we can associate a $\mathrm{PSL}(2,\mathbb{C})$ representation. This induces a regular map $\mathcal{D}(M,K) \to X(M)$. So we can study ideal points of $X(M)$ by studying ideal points of $\mathcal{D}(M,K)$. An ideal triangulation defines a matrix $(r'_{i,\nu}, r''_{i,\nu})$. I defined the *degeneration vectors* associated with the matrix $(r'_{i,\nu}, r''_{i,\nu})$. The coefficients of a degeneration vector represent growth rate of variables z_i.

Theorem 0.1 *If all the coefficients of the degeneration vector are positive, there exists ideal points of* $\mathcal{D}(M,K)$.

The boundary slope corresponding to the ideal point can be easily calculated by linear algebra. I also give a condition that an ideal point of $\mathcal{D}(M,K)$ detected by our method to be an ideal point of $X(M)$, therefore we can find ideal points of $X(M)$.

I made a program to calculate the degeneration vectors of an ideal triangulated 3-manifold with torus boundary. By using the program, I computed boundary slopes of some non-Montesinos knots.

Finite surgeries on $(-2,p,q)$-pretzel knots This is a joint work with Futer, Ishikawa, Mattman and Shimokawa. In [3], we showed that non-trivial Dehn surgeries on $(-2,p,q)$-pretzel knots do not produce manifolds with finite fundamental group with p, q odd and $5 \le p \le q$. Combining with Mattman's result, we showed that (p,q,r)-pretzel knot does not admit non-trivial finite Dehn surgeries unless $(-2,3,7)$, $(-2,3,9)$. We used the following methods: Agol-Lackenby's 6-theorem to a link which produces $(-2,p,q)$-pretzel knots, constructing a surjective map to a infinite group, estimate the Culler-Shalen norm of some $(-2,p,q)$-pretzel knots.