

Research plan

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I study the knot cobordism group and a slice-ribbon knot as its identity element, and finite type invariants of 2-bridge knots.

Slice-ribbon knots and knot cobordism invariants

1. I want to have a "mutation-sensitive" cobordism invariant: Recently we have two important cobordism invariants: One is given by J. Rasmussen and the other is given by P. Ozsvath and Z. Szabo. Rasmussen gave a cobordism invariant by using the Khovanov homology which is a homology-based extension of the Jones polynomial. Ozsvath and Szabo gave a cobordism invariant by using the Floer homology which is a homology-based extension of the Alexander polynomial. One of the remarkable differences between the Khovanov homology and the Floer homology is that the former is invariant under knot mutation but the latter is not. For instance, the Khovanov homology for the Kinoshita-Terasaka knot and the Conway knot are same, but the Floer homology for them is different. (They did not investigate the property for other mutants.) However, both cobordism invariants are invariant under knot mutation. On the other hand, I found that the seven candidates of the unidentified ribbon knots are all mutants or mutants-like. I think it seems a key for studying knot cobordism group and slice-ribbon knots to have a mutation-sensitive cobordism invariant.
2. I want to have the complete "ribbon knot atlas" as one of the basic works of knot theory.

Finite type invariants of 2-bridge knots

1. I want to derive some arithmetic results for two formulas for the Casson knot invariant of 2-bridge knots mentioned in my research statement.
2. I want to give a geometric interpretation for results on finite type invariants of $PSL(2, Z)$ mentioned in my research statement. Furthermore, I want to extend this work to a general mapping class group.
3. Many works on 2-bridge knots are done by using Conway's normal form. The map for the Gauss diagram of Schubert's normal form mentioned in my research plan worked very well. By using this map, some problems relating to Schubert's normal form may become easier. In fact, studying finite type invariants of 2-bridge knots seems to be the case. So I study finite type invariants of 2-bridge knots by using the Gauss diagram of Schubert's normal form and I derive some arithmetic results by using two formulas for them; one is a formula by means of a continued fraction expansion mentioned in my research statement and another is a formula without using a continued fraction expansion.