Plan of my research

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My concrete research plan consists of three parts as follows;

(1)Complementary Regions of diagrams of a knot, link and spatial graphs.

We would like to extend the result in [9] to spatial graph diagrams. Given a diagram of a knot or link, it can be regarded as a planar 4-valent graph embedded on the 2-sphere $\mathbb{S}^2$ by ignoring which strand is the overstrand at each crossing. This graph divides the sphere into n-gons, which we call faces and which meet at vertices and along edges. In [9], we studied the possibilities for the collection of complementary n-gon faces associated to the diagrams of a knot or link. By Euler Characteristic and the fact that a diagram D of a knot or link can be regarded as a planar 4-valent graph in $\mathbb{S}^2$, we have:

$$2p_2(D) + p_3(D) = 8 + p_5(D) + 2p_6(D) + 3p_7(D) + \cdots . \quad (*)$$

where $p_n(D)$ is the number of faces of D with i sides. A diagram of a spatial graph also can be regarded as a planar graph embedded on $\mathbb{S}^2$ by ignoring which edge is the overstrand at each crossing. However we can not use $(*)$ unless the spatial graph is an 4-regular graph embedded in $\mathbb{S}^2$. If we try to extend the result in [9] to spatial graphs, another equation like $(*)$ that can adjust to a diagram of a spatial graph is necessary. First of all, we examine spatial graphs which are embedded 4-valent graphs. We also examine 3-connected graphs which it is comparatively easy to treat.

(2) The characterization of knots realizing (3,4)-diagrams.

Though we found some universal sequence whose length is three, we have not found a universal sequence whose length is two. However there are infinitely many knots realizing (3,4)-diagrams. Hence we would like to know whether the sequence (3,4) is universal or not. So far, the way to prove that the given sequence is not universal has not been found except for using $(*)$. We would like to find knot invariants from which we can obtain infomation of complementary regions of diagrams. If (3,4) is not universal, we would like to characterize the knots realizing (3,4)-diagrams. We will also investigate whether the sequence (3,4) is universal from graph theoretical viewpoint.

(3) On an infinite sequence of mutually non-conjugate braids which close to the same knot

By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed n-braid ($n = 1, 2$ or 3) has only a finite number of conjugacy classes of n-braid representatives in the n-braid group B_n. H. R. Morton and T. Fiedler gave infinite sequences of 4-braids which result in the unknot and E. Fukunaga gave such a sequence for $(2, p)$-torus links. In [4], [5] and [7], we construct some such sequences. Since our method doesn't work for some links, we would like to improve it so that it can be used for links. In [5], generalizing sequences given by Morton, Fiedler and Fukunaga, for any knot K represented as a closed n-braid ($n \geq 3$), we gave an infinite sequence of mutually non-conjugate $(n+1)$-braids representing K. Though all braids in the infinite sequence given in [5][7] are reducible, we conjectured that for knots which is a closed n-braid satisfying certain conditions have such infinite sequence. Since we gave the partial answer for the conjecture in [4], we would like to solve the conjecture.