Abstract for our study

1. Our study on line configurations in complex planes

Let $l = l_1 \cup l_2 \cup \cdots \cup l_\mu$ be a collection of straight lines in $\mathbf{R}^2$. Each line is of the form $l_i = \{(x, y) \in \mathbf{R}^2 | a_i x + b_i y + c_i = 0\}$. Let $L_i = \{(x, y) \in \mathbf{C}^2 | a_i x + b_i y + c_i = 0\}$ and we have $L = L_1 \cup L_2 \cup \cdots \cup L_\mu$ in $\mathbf{C}^2$. This is called a real line configuration in $\mathbf{C}^2$. The complex projective plane $\mathbf{CP}^2$ is the quotient space of $\mathbf{C}^3 - \{\mathbf{0}\}$ by identifying (x, y, z) and $\lambda(x, y, z)$ for complex numbers x, y, z and $\lambda \neq 0$. Let $\mathcal{L}_i = \{[x, y, z] \in \mathbf{CP}^2 | a_i x + b_i y + c_i z = 0\}$ and we have $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \cdots \cup \mathcal{L}_\mu$ in $\mathbf{CP}^2$. This is called a real line configuration in $\mathbf{CP}^2$.

First we describe our study on a real line configuration $\mathcal{L}$ in $\mathbf{CP}^2$. We proved the following results concerning the first Betti numbers of abelian coverings of $\mathbf{CP}^2$ branched over real line configurations:

(1) An estimate of the first Betti numbers .

(2) A characterization of a central and general position line configurations in the terms of the first Betti numbers of abelian coverings.

(3) The first Betti numbers of the abelian coverings of the real line configurations up to 7components.

Next we describe our study on a real line configuration in $\mathbf{C}^2$. For a real line configuration L, we construct a ribbon surface-link which has the same group as L. If L is a central or general position line configuration, the genus of the constructed ribbon surface-link is the smallest of all the genera of the ribbon surface-links with the same group as L.

2. Our study on links in the three dimensional sphere

First we describe our study on 2 component links. We give a formula to express the first homology groups of the $\mathbf{Z}_2 \oplus \mathbf{Z}_2$ branced coverings of $L = K_1 \cup K_2$ in terms of those of three smaller cyclic branched coverings.

Next we describe our study on a table of manifolds. A. Kawauchi defined a well-order on the set of links, which induces a well-order on the set of link exteriors, and which eventually induces a well-order on the set of 3-manifolds. In fact, he enumerated the first 28 prime links, the first 26 prime link exteriors and the first 26 closed connected orientable 3-manifolds. We extended the prime link table from 28 to 443, the prime link exterior table from 26 to 399 and the manifold table from 26 to 133.