up date 14 March 2010

The Results of our Researches (Nobutaka Boumuki)

We carry out some researches into the following two subjects (I) and (II):

(I) Isotropic submanifolds of Riemannian symmetric spaces;
(II) Adjoint orbits of semisimple Lie groups.

We would like to explain about the subjects (I) and (II), separately.

(I): B.O'Neill (Canad.J.Math., 1965) has introduced the notion of isotropic immersion. Totally umbilic immersions or totally geodesic immersions are isotropic (but not vice-versa). Hence, one may regard the notion of isotropic immersion as an extention of the notion of totally umbilic or totally geodesic immersion. Now, we notice the following fact:

Fact: All parallel immersions $f : M \to N$ are isotropic immersions, but not vice-versa. Here we denote by M, and N any compact symmetric space of rank one, and any space form, respectively.

Taking the above Fact into consideration, we grapple with the following problem:

Problem: What is a sufficient condition for isotropic immersions $h : M \to N$ to be parallel ?

In the papers [5], [7], [9], [10], [12], [13] and [14], we settle the above Problem.

(II): Let G be a connected semisimple Lie group, and let $\mathfrak{g}$ denote its Lie algebra. Then G acts on $\mathfrak{g}$ via the adjoint representation Ad of G, and the orbit $\mathrm{Ad}(G)x$ is called the adjoint orbit of G through x, for an element $x \in \mathfrak{g}$. It is known that the adjoint orbit $\mathrm{Ad}(G)x$ admits a structure of a symplectic homogeneous G-manifold, and that each symplectic homogeneous G-manifold M can be expressed, up to covering, as the adjoint orbit $\mathrm{Ad}(G)y$ of G through some element $y \in \mathfrak{g}$. Therefore, one can conclude that symplectic homogeneous G-manifolds are related with the adjoint orbits of G.

In the paper [4] we give a method for determining Kaehler or pseudo-Kaehler homogeneous G-manifolds in terms of the root theory. In the papers [1] and [3] we study affine symmetric spaces G/H from a viewpoint of adjoint orbits, in the case where G/H admit structures of symplectic homogeneous G-manifolds.

References

[1] T. Noda and N. Boumuki, *On relation between pseudo-Hermitian symmetric pairs and para-Hermitian symmetric pairs*, Tohoku Mathematical Journal vol. 61, No. 2 (2009), pp. 67–82.

[2] N. Boumuki and T. Noda, *Decompositions of symplectic structures* (in Japanese), RIMS Kokyuroku 1623 (2009), pp. 89–101.

[3] N. Boumuki, *The classification of simple irreducible pseudo-Hermitian symmetric spaces: from a viewpoint of elliptic orbits*, Memoirs of the Faculty of Science and Engineering Shimane University vol. 41 (2008), pp. 13–122.

[4] N. Boumuki, *Isotropy subalgebras of elliptic orbits in semisimple Lie algebras, and the canonical representatives of pseudo-Hermitian symmetric elliptic orbits*, Journal of the Mathematical Society of Japan vol. 59, No. 4 (2007), pp. 1135–1177.

[5] N. Boumuki, *Isotropic immersions and parallel immersions of Cayley projective plane into a real space form*, New Zealand Journal of Mathematics vol. 36, (2007), pp. 139–146.

[6] N. Boumuki and S. Maeda, *Study of isotropic immersions*, Kyungpook Mathematical Journal vol. 45, No. 3 (2005), pp. 363–394.

[7] N. Boumuki, *Isotropic immersions of rank one symmetric spaces into real space forms and mean curvatures*, Contemporary Aspects of Complex Analysis, Differential Geometry and Mathematical Physics (ed. S. Dimiev and K. Sekigawa), World Scientific Publishing (2005), pp. 31–40.

[8] N. Boumuki, *Symplectic homogeneous spaces and adjoint orbits* (in Japanese), RIMS Kokyuroku 1460 (2005), pp. 1–10.

[9] N. Boumuki, *Isotropic immersions and parallel immersions of space forms into space forms*, Tsukuba Journal of Mathematics vol. 28, No. 1 (2004), pp. 117–126.

[10] N. Boumuki, *Isotropic immersions with low codimension of complex space forms into real space forms*, Canadian Mathematical Bulletin vol. 47, No. 4 (2004), pp. 492–503.

[11] N. Boumuki, *Remarks on real Lie groups with a complex Lie algebra*, Far East Journal of Mathematical Sciences vol. 13, No. 2 (2004), pp. 173–179.

[12] N. Boumuki, *Isotropic immersions of complex space forms into real space forms and mean curvatures*, Bulletin of the Polish Academy of Sciences Mathematics vol. 52, No. 4 (2004), pp. 431–436.

[13] N. Boumuki, *Isotropic immersions with low codimension of space forms into space forms*, Memoirs of the Faculty of Science and Engineering Shimane University vol. 37, (2004), pp. 1–4.

[14] N. Boumuki, *Characterization of parallel immersions of real space forms into real space forms* (in Japanese), RIMS Kokyuroku 1346 (2003), pp. 131–137.