Results of study

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We are interested in isoparametric hypersurfaces in spheres with four distinct principal curvatures. We expect that every isoparametric hypersurface in spheres with four distinct principal curvatures is related to moment map.

A hypersurface in the spheres S^n is called an *isoparametric hypersurface* if this one is a level set of an isoparametric function on S^n. It is known that isoparametric functions on S^n are obtained by the restrictions on S^n of Cartan-Münzner polynomial functions $\varphi : \mathbb{R}^{n+1} \to \mathbb{R}$.

Let G/K be an irreducible Hermitian symmetric space of compact type and of rank two. Then we get two invariant functions under the isotropy representation of G/K. One comes from a Cartan-Münzner polynomial. It is known that a principal orbit of the isotropy representation of G/K is a homogeneous isoparametric hypersurface in a sphere. By the definition of isoparametric hypersurfaces, there exists an isoparametric function on the sphere. This function is the restriction of a Cartan-Münzner polynomial φ. This polynomial φ is invariant under the isotropy representation of G/K. Another invariant function comes from a moment map. Since G/K is Hermitian, the isotropy representation of G/K is a Hamiltonian action. Thus, there exists a moment map μ for this action. By definition, μ is equivariant under the action. Hence, a composition function of μ and a norm which is invariant under the isotropy representation of G/K is invariant under the action.

Let φ be a Cartan-Münzner polynomial obtained from the isotropy representation of a Hermitian symmetric space G/K of rank two. Then, there exists a K-invariant norm on $\mathfrak{k}^*$ such that φ coincides with the squared-norm of the moment map μ for the isotropy representation of G/K.

Cartan-Münzner polynomials which we got are essentially the same as ones which are computed in Ozeki-Takeuchi (1976). The difference of both is a computational method.