RESEARCH ACHIEVEMENTS

IN DAE JONG

Alexander polynomials of alternating knots: The Alexander polynomial is a Laurent polynomial invariant for knots, which is characterized by two algebraic conditions. There are some studies on the Alexander polynomials of alternating knots, but the characterization is not achieved. In particular, the "trapezoidal conjecture" for the Alexander polynomials of alternating knots, which was originally proposed by Fox in 1960's, is still open. In the paper [1], I show the trapezoidal conjecture is true for alternating knots up to genus two.

Furthermore, in the paper [2], I give a family of linear inequalities on the coefficients of the Alexander polynomials of alternating knots of genus two. My approach is independent of Ozsváth-Szabó's approach using the Knot Floer homology to study the Alexander polynomials of alternating knots. (Ozsváth-Szabó's inequality for alternating knots of genus two is obtained as a consequence of my result.)

Gromov hyperbolicity and a variation of the Gordian complex: One of the problems we are interested in is to reveal properties of spaces which consists of the knots. However the space of knots (e.g. Gordian complex) is very "complicated", and thus it is hard to study them. In the paper [5] (with Kazuhiro Ichihara), we introduce a variation of the Gordian complex by using knot invariants, and show that the simplicial complex defined by using the Alexander polynomial and the Delta-move is Gromov hyperbolic.

Positive knots of genus two: We focus on the two properties of knots, that is the "positivity" of knots and the "alternatingness" of knots. In particular, in the paper [4] (with Kengo Kishimoto), we show that positive knots up to genus two is positive alternating or almost positive-alternating. We also show that positive knots up to genus two is quasi-alternating.

Cyclic and finite surgeries: A Dehn surgery is an operation to create a new 3–manifold form old one by using a knot. It is well-known due to Thurston that Dehn surgeries on a hyperbolic knot yield only finitely many non-hyperbolic 3–manifolds. Such Dehn surgeries are called exceptional surgeries. On knots in the 3–sphere, it is an interesting problem to determine all non-trivial Dehn surgeries which produce 3–manifolds with cyclic or finite fundamental groups, which we call cyclic surgeries / finite surgeries respectively. Here a cyclic surgery and a finite surgery are exceptional. In the paper [3] (with Kazuhiro Ichihara), we determine all cyclic and finite surgeries on Montesinos knots.

Seifert fibered surgeries: Among exceptional surgeries on Montesinos knots, detecting Dehn surgeries yielding Seifert fibered 3–manifolds with infinite fundamental groups is just remaining problem. In the paper [7] (with Kazuhiro Ichihara and Shigeru Mizushima), we determine Seifert fibered surgeries on alternating Montesinos knots.