Results of my research

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Pre-Bloch invariant Neumann and Yang defined the Bloch invariant for a finite volume hyperbolic 3-manifold. In [1], I defined the pre-Bloch invariant for a compact 3-manifold with boundary whose genus is greater than one. Since finite volume hyperbolic 3-manifolds have toral boundary, this is a generalization of the invariant of Neumann-Yang. The invariant is defined for a fixed pants decomposition on the boundary. For each boundary curve γ_i of a pair of pants, I defined a complex parameter H_i by the square of the eigenvalue of $\rho(\gamma_i)$. For γ_i, I also defined the twist parameter which measures how the two pair of pants adjacent to γ_i are glued. By using these parameters, I a variation formula of the volume function.

A method for finding ideal points Yoshida introduced a method for finding ideal points from an ideal triangulation of a 3-manifold. His method gives necessary conditions which the valuation associated to an ideal point satisfies. Since the conditions are given by linear equations, we can find candidates of ideal points by solving linear equations. But they might not correspond to ideal points. In fact, there are many solutions of the linear equations which do not correspond to ideal points. In [2], I gave a criterion for a candidate to correspond actually to an ideal point. I also showed how to compute the number of ideal points corresponding to the candidate. In my method, we only have to compute determinants of some matrices to find ideal points.

I wrote a program to carry out my method for an ideally triangulated 3-manifolds with torus boundary. By using the program, I computed boundary slopes of some non-Montesinos knots. I found the following examples: alternating knots which have odd boundary slopes (all alternating Montesinos knots have only even boundary slopes) and alternating knots which have non-integral slopes, knots whose diameters of boundary slopes are greater than twice the number of crossings.

Finite surgeries on $(-2,p,q)$-pretzel knots This is a joint work with Futer, Ishikawa, Mattman and Shimokawa. In [3], we showed that non-trivial Dehn surgeries on $(-2,p,q)$-pretzel knots do not produce manifolds with finite fundamental group with p, q odd and $5 \leq p \leq q$. Combining with Mattman's result, we showed that (p,q,r)-pretzel knot does not admit non-trivial finite Dehn surgeries unless $(-2,3,7)$, $(-2,3,9)$. We used the following methods: Agol-Lackenby's 6-theorem to a link which produces $(-2,p,q)$-pretzel knots by Dehn surgeries, constructing a surjective map to a infinite group, estimate of the Culler-Shalen norm of some $(-2,p,q)$-pretzel knots.

Quandle homology and knot invariants This is a joint work with Ayumu Inoue. We gave a construction of quandle cocycles from a geometrical view point. We defined a simplicial version of quandle homology and constructed a homomorphism from the usual quandle homology to the simplicial one. As a result, we showed that the n-dimensional hyperbolic volume is a quandle cocycle of the quandle consisting of parabolic elements of $\mathrm{Isom}^+(\mathbb{H}^n)$. In the case of $\mathrm{Isom}^+(\mathbb{H}^3) \cong \mathrm{PSL}(2,\mathbb{C})$, we can also show that the Chern-Simons invariant is also a quandle cocycle. In fact we constructed a map from quandle homology of the quandle consisting of parabolic elements of $\mathrm{PSL}(2,\mathbb{C})$ to the extended Bloch group. For a knot in S^3 with a parabolic representation, we can associate a quandle cycle which is a conjugacy invariant of the representation. Clearly the image under the map is an invariant with values in the extended Bloch group, moreover we showed that the image coincides with the invariant defined by Neumann. As a result we obtain a diagrammatic description of the hyperbolic volume and Chern-Simons invariant of knot complements.