My results (Teruhisa Kadokami)

1. "Seifert complex for links and 2-variable Alexander matrices"

D. Cooper considered a union of Seifert surfaces for the components of a link where the surfaces may intersect one another and we can restrict the intersections as clasp singularities. Then he defined the union of the surfaces as a *C-complex* for the link, and studied for the 2-component case. He also defined an *R-complex* (resp. an *RC-complex*) as the case that the intersections are only ribbon singularities (resp. ribbons and clasps). We named the union of the surfaces as a *Seifert complex*. We confirmed the method of Cooper and characterized the 2-variable Alexander matrices for 2-component links. By using the result, we reproved both the Torres formula for 2-component links and the characterization of the Alexander polynomial for 2-component algebraically split links due to L. Bailey and Y. Nakanishi.

2. "Proper link, algebraically split link and Arf invariant" (with Akira Yasuhara)

We defined a new Arf invariant for an R-complex of an algebraically split link. I contributed to the work by showing well-definedness of the definition. The ordinary Arf invariant is defined for a proper link, and it is an link invariant. It has additivity under connected sums for boundary links, not for algebraically split links. On the other hand, our new Arf invariant is an invariant of a pair of an R-component and its component surfaces, and it has additivity under connected sums.

3. "Component-isotopy of Seifert complexes"

D. Cooper showed fundamental moves among 2-component C-complexes for the same link. I extended the result to the n-component C-complex case. Then I introduced a new move, and pointed out that it cannot be removed by showing an example that certain two 3-component C-complexes for the Borromean rings cannot be transformed each other without using the move. I also proved that there are fundamental moves for singular Seifert surfaces of a knot.

4. "Detecting non-triviality of virtual links"

I proved that any flat virtual link (*projected virtual link* in the paper) has a *reduced diagram* and two reduced diagrams can be transformed each other by a finite sequence of the third Reidemeister moves and interchanging components. We can show the non-triviality of many virtual links including Kishino's knot by using the facts.

5. "Classification of closed virtual 2-braids"

I classified completely closed virtual 2-braids. I proved it by using the surface bracket polynomial due to H. Dye and L. Kauffman, and a certain subgroup of the 3-string braid group.

6. "Connected sum and prime decomposition of virtual and flat virtual links"

In general, the connected sums of virtual links are not uniquely determined. We introduced a new concept *pointed virtual link* which is a pair of a virtual link diagram and finite points on it under a certain equivalence relation, and defined the connected sum of pointed virtual links by connecting along the points. When we connect some virtual/flat virtual links, the sum is performed along a *connecting tree*. By defining an equivalence relation for connecting trees suitably, prime factors and the decomposition are uniquely determined.

7. "Reidemeister torsion of homology lens spaces"

Since any homology lens space can be obtained by p/q-surgery along a knot in an integral homology 3-sphere where p and q are coprime and $|p| \geq 2$, we can compute the Reidemeister torsions of it by surgery formulae. We studied the conditions for the Reidemeister torsions of a homology lens space to be the same as that of a lens space when the Alexander polynomial of the knot is (1) the same as that of a torus knot, (2) of degree two, and (3) the same as that of a $(-2, m, n)$-pretzel knot (with Yuichi Yamada). We obtained the following: (1) necessary and sufficient conditions, (2) the Alexander polynomial should be of the form $t^2 - t + 1$, and (3) speciality of the $(-2, 3, 7)$-pretzel knot. Moreover (4) we characterized the form of the Alexander polynomial for the knot above (with Yuichi Yamada), (5) we applied the similar method for Seifert surgeries, and we determined lens surgeries along (6) the Whitehead link (with Masafumi Shimozawa and Noriko Maruyama), (7) certain two 2-component links which are discussed well in an area of generalized rational blow down (with Yuichi Yamada), and (8) the Milnor links.

8. "Iwasawa type formula for covers of a link in a rational homology sphere" (with Yasushi Mizusawa)

The Iwasawa formula in Number Theory to compute the order of the ideal class group for a certain ramified extension of an algebraic field. It corresponds the formula in Knot Theory to compute the order of the first homology group for a certain branched abelian covering of an integral homology 3-sphere due to R. Fox, J. Mayberry-K. Murasugi and J. Porti. We extended the latter result to the case of a branched cyclic covering for a rational homology 3-sphere, and exhibited some examples.

9. "An estimation of the C_k-unknotting number for a C_k-trivial link" (with Akira Yasuhara)

We defined a trivial link up to C_k-moves as a *C_k-trivial link*, and the minimal number of C_k-moves to deform the link into the trivial link as the *C_k-unknotting number* of the link. We obtained an upper bound of it from Milnor's μ-invariant and the clasper theory. More strict estimations can be obtained for the cases $k = 2$ and 3. Moreover we made a table of the C_2-unknotting numbers of C_2-trivial links with the crossing number up to ten.

10. "An integral invariant from the knot group" (with Zhiqing Yang)

We defined the minimal number of normal generators of the commutative subgroup for a knot group as the *MQ index* and studied its properties. (1) The Nakanishi index is a lower bound of it, and both the rank -1 of the knot group and the tunnel number are upper bounds of it. (2) The MQ index of the connected sum of the $(2, p)$-torus knot and the $(2, q)$-torus knot is one if and only if $\gcd(p, q) = 1$. That is, the MQ index has not additivity.

11. "Alexander polynomials of algebraically split amphicheiral links"

We obtained necessary conditions for the Alexander polynomials of algebraically split amphicheiral links.