Research Plan

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1. **On a boundary of the Bers fiber space and a boundary of the Teichmüller space**

   Let G be a torsion free finitely generated Fucshian group of the first kind acting on the upper half plane U. Assume that U/G is a Riemann surface of genus g with n punctures.

   The Teichmüller space $T(G)$ of G is embedded into the complex vector space $B_2(L, G)$ of holomorphic automorphic forms of weight -4 on the lower half plane L with respect to G. We identify the image of $T(G)$ under the embedding with $T(G)$, then the boundary $\partial T(G)$ of $T(G)$ is naturally defined.

   The fiber space $F(G)$ over $T(G)$ whose fiber is a quasidisk is defined. By the embedding as above, we see that $F(G)$ becomes a domain in $B_2(L, G) \times \mathbb{C}$. Now let $\dot{G}$ be another Fuchsian group and $U/\dot{G} \to U/G - \{\text{a point}\}$ be a conformal bijection. Then Bers showed there exists an isomorphism of $F(\dot{G})$ onto $T(G)$.

   So by means of this isomorphism and the theory of Thurston's compactifiaction of Teichmüller spaces, I shall study a correspondence of a boundary of $F(\dot{G})$ and $\partial T(G)$.

2. **On holomorphic families of Riemann surfaces**

   Let B be a hyperbolic Riemann surface and suppose a holomorphic family of Riemann surfaces of type (g, n) over B is given, where g is the number of genus of a fiber and n is the number of punctures of the fiber.

   Then we have a holomorphic map from Δ (the universal covering surface of B) to the Teichmüller space of type (g, n).

   If the first research in **1** develops, then I expect to have a correspondence of $\partial \Delta$ and $\partial T_{(g,k)}$. From this, we see a detailed information about holomorphic families.

3. **On extensions of holomorphic motions**

   Holomorphic motions were introduced by Mañé, Sad and Sullivan. They have been important in the study of dynamical system and Kleinian groups. Bers and Royden showed the intimate relationship between Teichmüller spaces and holomorphic motions.

   Yunping Jiang and Sudeb Mitra showed many results about extensions of holomorphic motions over Teichmüller spaces.

   I will analyze their results and apply them to a study of Bers fiber spaces.