Plan of my research

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My concrete research plan consists of three parts as follows;

(1) On combinatorial properties of knot and link diagrams.

There are knots defined by the properties of their diagrams like alternating knots. And a lot of properties of alternating knots are known. By focusing on the complementary regions, we try to find the properties of diagrams which reflect properties of knots and links like 'alternating'. In [8], we defined universal sequences and gave some universal sequences. However a universal sequence with length two is not found so far. We conjecture that the sequence (3,4) is not universal. We would like to solve this conjecture affirmatively and characterize the knots and links which have diagrams each of whose regions is a triangle or a quadrangle.

Since a way to proof given sequence is not universal has not been found except for using Euler characteristic, we try to find knot invariants from which we can obtain information of complementary regions of diagrams. We are planning the approach from graph theoretical viewpoint.

(2) Complementary Regions of diagrams of spatial graphs.

Given a diagram of a knot or link, it can be regarded as a planar 4-valent graph embedded on the 2-sphere $\mathbb{S}^2$ by ignoring which strand is the overstrand at each crossing. This graph divides the sphere into n-sided regions, which we call faces and which meet at vertices and along edges. In [8], we studied the possibilities for the collection of complementary n-sided regions associated to the diagrams of a knot or link. The result concerning about odd sided regions can be extended to diagrams of spatial embedding of planar graphs. Hence we try to extend these result in [8] to spatial graph diagrams. Moreover, we want to find properties of the diagrams of spatial graphs which we mentioned in (1).

(3) On an infinite sequence of mutually non-conjugate braids which close to the same knot

By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed n-braid ($n = 1, 2$ or 3) has only a finite number of conjugacy classes of n-braid representatives in the n-braid group B_n. H. R. Morton and T. Fiedler gave infinite sequences of 4-braids which result in the unknot and E. Fukunaga gave such a sequence for $(2, p)$-torus links. In [1], [2] and [6], we construct some such sequences. In [1], generalizing sequences given by Morton, Fiedler and Fukunaga, for any knot K represented as a closed n-braid ($n \geq 3$), we gave an infinite sequence of mutually non-conjugate $(n+1)$-braids representing K. A. Stoimenow extend the result to links satisfying certain conditions. We conjectured that for knots (or links) which are closed n-braids satisfying certain conditions have such infinite sequences. In a joint project with A. Stoimenow, we plan to give solution of the conjecture.