Research Plan

Shunsuke Yamana

One of the most surprising aspects of the classical theory of modular forms is the wealth of information carried by the Fourier coefficients. For example, the Fourier coefficients of elliptic modular forms play a fundamental role in number theory. In the case of half-integral weight, the situation is somewhat more interesting; the square of their Fourier coefficient is related to a certain special value of L-functions, where the sign is still mysterious. In the case of modular forms in several variables, the investigation of the Fourier coefficients of theta functions gives many deep results on quadratic forms.

On the other hand, these modular forms can be viewed as smooth vectors in certain representations of the ambient group on spaces of functions on that group invariant under its discrete subgroups. This leads to the general notion of modular forms, and then to the study of the underlying representations, what we call automorphic representations. One of principal issues raised by Langlands is functoriality, the relation between automorphic representations on different groups. Functoriality was used in Wiles's proof of Fermat's last theorem and gained recent importance.

We studied holomorphic modular forms by investigating their Fourier coefficients so far. However, some of our works are related to functoriality, and studying functoriality requires the insights of representation theory. In the immediate future, I would like to study automorphic forms mainly by the classical and representation theoretic methods.

We are planning to study the following problems.

1. **Some extensions of the Siegel-Weil formula for quaternionic unitary groups**
   We extend the Siegel-Weil formula to the case of quaternion dual pairs beyond the range of absolute convergence. The failure of the Hasse principle for skew-hermitian forms over quaternion skew fields makes our extension in the quaternion case somewhat interesting.
2. **Local factors of representations for classical groups**
   It is believed that there are natural invariants of irreducible representations which coincide with the factors of Artin type associated to the relevant representations on the Galois side under the local Langlands correspondence. For example, these are L and epsilon factors. The theory of these invariants has not been developed satisfactorily in spite of their importance in representation theory. We would like to have a general theory of local factors due to applications in representation theory and arithmetic problems.
3. **A construction of the kernel function of the Ikeda lifting**
   We try to construct explicitly the holomorphic kernel function of the Ikeda lifting.
4. **Determination of modular forms by Fourier coefficients**
   We try to extend the result of our paper [1] to Hilbert modular forms or non-holomorphic modular forms. It seems interesting as well to consider the problems obtained by replacing determination of modular forms with algebraicity or integrality of modular forms.