これまでの研究成果のまとめ（英訳）

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I have studied wavelet theory and its application to some important function spaces in real analysis so far. In addition I have grappled with problems on giving another expression of the norms of those spaces in terms of wavelet coefficients and constructing bases with nice properties. Recently approximation theory with bases given by wavelets is applied and marked in many areas, for example image analysis, signal analysis, statistical estimation and numerical analysis of partial differential equations. In order to apply wavelet theory on various function spaces more effectively we naturally need to investigate themselves deeply. I have also studied both application of wavelets and boundedness of several operators including singular integrals and fractional integrals on function spaces.

I have noted the theory of A_p weights due to Muckenhoupt and studied weighted function spaces so far. In particular I have investigated some properties of the class of A_p^{loc} weights which is a generalized A_p class obtained by localization. The interest in A_p^{loc} is that it includes exponentially increasing weights which we cannot treat in the context of A_p. I have constructed characterizations and unconditional bases of some weighted function spaces with A_p^{loc} weights in terms of wavelets by applying properties of the weights. I have additionally proved that bases called “greedy” can be constructed on weighted Sobolev spaces and weighted Triebel–Lizorkin spaces. The greedy bases have excellent properties in approximating functions.

On the other hand, I have mainly studied function spaces with variable exponent in recent years. The spaces are being watched with keen interest not in real analysis but also in partial differential equations and in applied mathematics because they are applicable to the modeling for electrorheological fluids and image restoration. The theory of function spaces with variable exponent has rapidly made progress in the past twenty years and a new area called “Variable Exponent Analysis” is being established. I have understood that variable exponents have some properties similar to A_p weights applicable to various analyses in the process of studies. Noting that, I have first given characterization of Lebesgue and Sobolev spaces with variable exponent which are the most fundamental function spaces with variable exponent by virtue of wavelets. I have additionally constructed unconditional bases on those spaces. I have also generalized the Herz space that is one of the most important function spaces in real analysis using variable exponent and defined a new function space named “the Herz space with variable exponent”. Herz spaces are arising from the study on characterization of multipliers on the classical Hardy spaces. Compared with the usual Lebesgue space, we see that a Herz space has an interesting norm which represents markedly both global and local properties of functions. I have proved boundedness of general subliner operators including the Hardy–Littlewood maximal operator and singular integrals on Herz spaces with variable exponent provided proper restricted conditions for exponents. Applying the result I have showed that we can construct characterization and unconditional bases of the spaces in terms of wavelets. I have additionally proved boundedness of commutators with BMO functions and singular integrals or fractional integrals on the spaces.

I have also studied applications of wavelets to complex analysis and obtained two results on holomorphic function spaces of one variable so far. One is a result on Bergman spaces on strip domains which gives an necessary and sufficient condition for the extension of a function belonging to $L^2(\mathbb{R})$ to one belonging to the Bergman space in terms of wavelet coefficients using a band-limited wavelet. The other result concerns Bargmann–Fock spaces on the complex plane. Noting a connection between the spaces and modulation spaces, I have constructed characterization and greedy bases of them using Gabor frames.