Research achievement (研究成果の英訳)

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We researched the large-time behavior of solutions to viscous conservation law in one dimension. The following is the central study results for me.

Generalization of flux

I think it is important to study the large-time behavior of solutions to viscous conservation law with non-convex flux, because many mathematical model of viscous conservation law has non-convex flux. For example, "Two phase flow problem, Etching of semiconductor and Night traffic flow problem" has non-convex flux. In 1997, Matsumura-Mei proved the asymptotic stability of viscous shock profile for a system of visco-elasticity with a non-convex nonlinearity. On the other hand, there was no research results for the asymptotic stability of superposition of stationary wave and rarefaction wave. Consequently, we showed in paper [1] the asymptotic stability of superposition by defining a new weight function and applied the weighted energy method introduced by Matsumura-Mei. Since our weight function is simple, it is applicable to reduce the convergence rate of solution as we see below.

Convergence rate

In paper [2], we derived the convergence rate of solutions to the initial-boundary value problem for scalar viscous conservation laws. We study not only the case where the flux is convex but also the case where the flux is in wider class. For the proof, we employ the weight function which is defined in [1], and combine L^p-L^1-estimate. Our weight function is applicable not only to the viscous conservation law with convex flux but also with non-convex flux.

Degenerate problem for Damped-wave equation

In 2008, Ueda-Kawashima suggested that the asymptotic behavior of the solution for Damped-wave equation is similar to one of viscous conservation law and they proved the asymptotic stability of stationary solution of Damped-wave equation. In [3], we extended the Ueda-Kawashima's result to the case where Damped-wave equation has a wide class of convection term. For the proof, we introduced a new weight function and successfully showed the asymptotic stability of the superposition of stationary solution and rarefaction wave. We further found out the fact that the sub-characteristic condition which had been assumed on all space is enough to be imposed only on the far field. Since the sub-characteristic condition on the all space is thought as necessary condition for the existence of the solution, our result set up a new problem for the existence of solution for the Damped-wave equation.

Non-degenerate problem for Damped-wave equation

In [4], we extend the result [3] and we showed that the asymptotic stability of non-degenerate stationary solution. We use anti-derivative method and weighted energy method. We also derive convergence rate by using space-time weighted energy method. As in paper [3], we also showed that the sub-characteristic condition is enough to be imposed only on the far field.