Research statement (project)

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Over the next few years I am planning to work about the following subjects.

To unify toric manifolds and toric HK manifolds from topological point of view. Tori HK manifolds themselves are quite different from toric manifolds as a space. However, similar phenomena often occurs between them: for example, their equivariant cohomology rings are isomorphic to the Stanley-Reisner rings of their corresponding combinatorial objects; or equivariant cohomological rigidity holds for both of them. Now I am studying the properties of the class which is some kind of topological generalization of toric HK manifolds defined in (15), under the motivation of "to unify toric and toric HK manifolds from topological point of view". Until now, I compute equivariant cohomology rings by using the main result of (14) (also see (31)). This study can be regarded as the geometric counterpart of the study of hypertorus graphs. Over the next few years, I would like to study them from both of geometry and topology more deeply, and finally I aim to find the quaternionic analogue of torus manifolds.

Extended actions of GKM manifolds. According to Wiemeler, in general, if torus manifolds have extended actions, then their transformation groups are SU or SO-types, i.e., groups whose root systems are A_ℓ, B_ℓ or D_ℓ types, such as (6), (9). So we are naturally led to ask the problem "what kind of classes have the extended actions with other types?" GKM manifolds could be one of such classes. Because all of the homogeneous spaces G/H with same ranks are contained in GKM manifolds, all root systems of type A_ℓ–G_2 can be appeared as the extended actions on GKM manifolds. Now, in (16), I am studying extended actions of GKM manifolds (also see (19)). Until now, I defined some class which may be called root systems of type A of GKM graphs. In this project, I aim to define root systems of all types of GKM graphs, and apply them to the study of extended actions of GKM manifolds (or to characterize GKM graphs from combinatorial point of view).

GKM $\mathbb{C}P$-towers. As is well known, the projectivization of $(n_2 + 1)$-dimensional vector bundles over $\mathbb{C}P^{n_1}$ yields $\mathbb{C}P^{n_2}$-bundles over $\mathbb{C}P^{n_1}$. Again the projectivization of $(n_3 + 1)$-dimensional bundles over these objects yields "$\mathbb{C}P^{n_3}$-bundles over $\mathbb{C}P^{n_2}$-bundles over $\mathbb{C}P^{n_1}$." $\mathbb{C}P$-tower is defined by iterating this construction. If each vector bundles can be decomposed into line bundles, then $\mathbb{C}P$-tower has the structure of toric manifolds and called generalized Bott tower. In general, they do not have structures of toric manifolds. However, they often become GKM manifolds. Now, in (17), I and Suh are studying the properties of $\mathbb{C}P$-tower. As I mentioned in "summary", we proved (equivariant) cohomological rigidity of 6-dimensional $\mathbb{C}P$-towers. In this year, we are planning to study more general cases. Moreover, flag manifolds are contained in $\mathbb{C}P$-towers. So, we can expect that our study could construct a bridge between the study of flag manifolds and the study of Bott towers (independent two areas). As the other goal of this project, I also aim to solve the open problem; the cohomological rigidity problem of Bott manifolds.

Relations with other areas via GKM graph. Recently, GKM graph is used in many areas (not only topology). For example, in mathematical physics, FTCY graph is defined to compute the Gromov-Witten invariant of toric Calabi-Yau 3-folds (the FTCY graph can be regarded as the special type of GKM graphs). In representation theory (algebraic geometry), Fiebig translated the Lustig's conjecture into the problem on GKM graphs and proved it. I believe that there must be many possibilities to connect between topology and other areas (in particular combinatorics). Although I still have to learn other areas, I assume I can contribute to find connections between seemingly unconnected fields of mathematics by using GKM graphs. In order to achieve that, I would like to discuss with mathematicians who study other areas, and share our idea.

I am looking forward to discussing with other mathematicians in OCAMI and having an opportunity to contribute to the development of the institute.