Research Statement (summary)

Shintarô Kuroki

My primary area of research is the study of transformation groups on manifolds from topological point of view. In particular, here, I would like to summarize my researches which lie in Toric Topology; though my research consists of three areas as I have already written in last year. Moreover, I would like to focus on the related topics about my research project. The numbers appearing below corresponds to that in "List of achievements".

Extended actions of torus actions on torus manifolds. A torus manifold is a $2n$-dimensional T^n-manifold with fixed points, e.g. the complex projective space $\mathbb{C}P$ or even dimensional sphere. This manifold is defined by Hattori-Masuda as an ultimate generalization of toric manifolds from topological point of view. This is now studied by several mathematicians, and regarded as one of the central objects in Toric Topology. Note that T^n is the maximum dimension which can act on $2n$-dimensional manifold effectively; therefore, this can be regarded as the very natural object from topological point of view.

It is important to study the extensions to non-abelian group actions of torus actions on torus manifolds, because this problem is related to find "most natural symmetries on torus manifolds". I classify in (6) that extended actions are transitive actions, and in (7) those are cohomogeneity one actions, i.e., there exist codimension one orbits. As a corollary of these classifications, we have that toric manifolds with transitive extensions are jut products of $\mathbb{C}P$'s and toric manifolds with cohomogeneity-one extensions are $\mathbb{C}P$-bundles over products of $\mathbb{C}P$'s.

Cohomological rigidity problems for several classes of manifolds. Cohomological rigidity problem is the problem which asks whether cohomology ring isomorphisms induce homeomorphisms on manifolds. In general manifolds, of course the answer is NO. However, if we restrict the class of manifolds, sometimes the answer is YES, or unknown, e.g., for toric manifolds this problem is still open. This is important problem because this is related to the classical and central problem in topology "to find the topological complete invariant of manifolds"

In (10), I and Choi classify topological types of some class $\mathcal{M}$ in the torus manifolds in (6), (9). We make a list of topological types of $\mathcal{M}$ by using cohomology rings and real characteristic classes. As a conclusion of this result, we give the subclass of $\mathcal{M}$ which satisfies the cohomological rigidity, and moreover we give counter examples of cohomological rigidity problems about torus manifolds which asked by Masuda-Suh.

A tori hyperKähler manifold, toric HK manifold for short, is the hyperKähler analogue of toric manifolds. This is a $4n$-dimensional space with T^n-action, e.g., cotangent bundle $T^*\mathbb{C}P$. I prove in (11) that equivariant cohomology distinguishes hyperhamiltonian structures on toric HK manifolds (this may be called equivariant cohomological rigidity). In (12), I proved that the set of $4n$-dimensional toric HK manifolds satisfies the cohomological rigidity.

Now I and Suh are studying $\mathbb{C}P$-towers defined by iterations of $\mathbb{C}P$-bundles (see "project" for detail s). In (17), we studied the (equivariant) cohomological rigidity of them.

GKM graph and gaph equivariant cohomology. A GKM manifold is a T^ℓ-manifold M^{2m} ($m \geq \ell$) such that the orbit type of less than one dimensional orbits has the structure of a graph. For example, manifolds appearing above are GKM manifolds. A GKM graph is defined by the graph, obtained by the orbit space of less than one dimensional orbits, labeled by tangential representations.

In (14), I define the class of GKM graphs called hypertorus graph, which contains the GKM graphs induced from the toric HK manifolds, and compute their graph equivariant cohomology rings abstractly. As a corollary, we have equivariant cohomology rings for wider class than the class of toric HK manifolds, e.g., manifolds defined in (15). In (16), I study combinatorial structures of some GKM graphs induced from GKM manifolds with $SU(n)$-extended actions (also see (30)). In particular, if there exists some class which may be called root systems of type A in graph equivariant cohomology, then their GKM graphs have the projection to the complete graph or after blowing up then we have such GKM graphs. These two graphs correspond to non-primitive manifold, i.e., manifolds isomorphic to $SU(n) \times_H N$, or primitive manifold, respectively, in geometry. In (17), I define the set of GKM graphs induced from 6-dimensional $\mathbb{C}P$-towers abstractly, and compute the graph equivariant cohomology rings.