Program of research

1. The study on curves in purely imaginary octonions $\mathrm{Im}\,\mathfrak{C}$.

   We determine the invariant functions of curves in $\mathrm{Im}\,\mathfrak{C}$ up to the action of G_2 and we prove a G_2-congruence theorem of curves in $\mathrm{Im}\,\mathfrak{C}$. As an application, we give examples of pairs of curves which are $SO(7)$-congruent to each other but not G_2-congruent. We give an explicit representation of G_2-frame field along helices in 3-dimensional and 4-dimensional Euclidinan spaces. We note that the G_2-invariant functions are not constant.

   In the similar way, we study on curves in in 5-dimensional, 6-dimensional and 7-dimensional Euclidinan spaces in $\mathrm{Im}\,\mathfrak{C}$.

2. The study on moduli space of induced almost complex structures on hypersurfaces of purely imaginary octonions $\mathrm{Im}\,\mathfrak{C}$.

   (1) Investigate the moduli space of induced almost complex structures on hypersurfaces of the product manifold $T^2 \times \mathbf{R}^4$ and its quotient $T^2 \times (\mathbf{R}^4/\Gamma)$ or $T^2/\Gamma \times \mathbf{R}^4/\Gamma$ etc.

   (2) Study on the relationship between the flat Klein bottle (which is obtain by C. Tompkins) and the generalized cylindrical helix. We note that we show that we can construct flat surface of $\mathbf{R}^4$ from the generalized cylindrical helix of $\mathbf{R}^4$. And that time, the generalized cylindrical helix is a geodesic in some flat surface. So we investigate the relationship between the flat Klein bottle and the generalized cylindrical helix.

   (3) Investigate the moduli space of the induced almost complex structures on hypersurfaces of cohomogeneity 1 in $\mathrm{Im}\,\mathfrak{C}$.