

I have two theme.

(1) Lorentzian Ricci solitons

N.Rahmani and S.Rahmani (2006) proved that any left-invariant Lorentzian metric on H_3 is classified into three types g_1 , g_2 and g_3 , up to isometry and scaling. They showed that g_2 has negative constant curvature, g_3 is flat and g_1 is not Einstein. Under such the background, we characterize the left-invariant Lorentzian metric g_1 as a Lorentzian Ricci soliton. Moreover, we treated the group $E(2)$ of rigid motions of Euclidean 2-space and the group $E(1,1)$ of rigid motions of the Minkowski 2-space. In particularly, we proved that $E(2)$ has a non-flat Lorentzian Ricci soliton. This leads to new examples of Lorentzian Ricci solitons without Riemannian analog. And I study Sol-soliton. We proved that there are Lorentzian Ricci solitons on the higher-dimensional classical Heisenberg group by using sol-soliton.

(2) Cohomogeneity one metrics

Many researchers studied what kind of condition a cohomogeneity one metrics with respect to a G -action becomes Einstein metric and the Ricci soliton. I studied an extension of 3-dimensional unimodular Lie groups with a Ricci soliton structure, and when the evolution in the extra 1-dimension moves like the Ricci flow, then this has a Ricci-flat metric (an Einstein metric with zero Ricci tensor). In other words, when the triple of functions $\{a(t), b(t), c(t)\}$ constructing the orbit satisfies the Ricci flow equation, we examined whether the resulting cohomogeneity one metric is a Ricci-flat metric. We have already proved that there are cohomogeneity one Ricci-flat metrics with respect to $E(2)$. Moreover I proved that these metrics have the Hyper-kähler structure.