Plan of Research

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In various occasions, it has been noticed that certain aspects of four dimensional quantum field theories resemble those of two dimensional QFTs. In particular, for theories with conformal symmetries, such as string theories and gauge theories. This has close connection with integrability.

Recently, a large class of $\mathcal{N} = 2$ superconformal $SU(n)$ quiver gauge theories in four dimensions are constructed by Gaiotto. Subsequently an interesting conjecture has been made by Alday, Gaiotto and Tachikawa (AGT). The AGT conjecture propose the equivalence of the Nekrasov partition function of the $SU(2)$ quiver gauge theory and the 2d conformal block of the Liouville theory. This conjecture is soon generalized to $SU(n)$ cases. It states that the Nekrasov partition functions of the $SU(n)$ quiver gauge theories can be expressed by using the conformal blocks of the 2d conformal Toda field theories. These are followed by a number of extensive checks and pieces of supporting evidence.

The Nekrasov partition function is a regularization of the instanton partition function of the gauge theory on the so-called Ω-background. It contains regularization parameters ϵ_1 and ϵ_2. At $\epsilon_1 = -\epsilon_2 = 0$, the leading part of the (logarithm of) partition function reproduces the Seiberg-Witten prepotential.

Itoyama and Oota observed that the Dotsenko-Fateev multiple integral representation of the conformal block can be interpreted as a β-deformed matrix model of the Selberg type and established a calculation method of the q-expansion using a formula associated with the Jack polynomials. Many matrix model calculations are performed at the large matrix size N limit. But our method allows to calculate the expansion coefficients at finite N. This leads, not to compare the matrix model curve with the Seiberg-Witten curve at large N, but to direct comparison of the matrix model partition function with the Nekrasov partition function. We have shown that for $SU(2)$ gauge theory with $N_f = 4$ flavors, by appropriately matching parameters, the matrix model correctly reproduces the Nekrasov partition function for some lower orders.

Then, Itoyama-Oota-Yonezawa considered the scaling limits of this matrix model which corresponds to the massive limits from $N_f = 4$ to $N_f = 3$ and subsequently to $N_f = 2$. The obtained expression corresponds to the Nekrasov partition functions for these cases or to the irregular conformal blocks.

Hence, we continue our investigation to consider the scaling limits to $N_f = 1$ and to $N_f = 0$. We will also study β-deformed matrix models which correspond to gauge theories with other types of gauge groups or to quiver gauge theories.

Furthermore, there are unsolved problems in our calculation method. We use the Jack symmetric polynomials which are homogeneous polynomials. But they are not good expansion basis in order to compare the Nekrasov partition function. We have shown that certain inhomogeneous symmetric polynomials provide a good basis for comparison. But their properties, definition etc, are still unclear. We would like to investigate these points.