The plan of our study

Every link can be realized as a closed braid. A. Kawauchi suggested the following project: enumerate the prime links by using the braid expressions and enumerate the closed connected orientable 3-manifolds by using the prime link table. Our main purpose is to make a table of the 3-manifolds according to this project.

We describe the outline of the project. A well-order (called a *canonical order*) was introduced on the set of links by A. Kawauchi [K] (see also A. Kawauchi and I. Tayama [KT1]).

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link and we call the number the *length* of the link. We note that a link L is smaller than a link L' in the canonical order if the length of L is smaller than that of L', and for any natural number n there are only finitely many links with lengths up to n. Let $\mathbf{L}^p$ be the set of prime links and $\mathbf{M}$ the set of closed connected orientable 3-manifolds. Let $\chi : \mathbf{L}^p \to \mathbf{M}$ be a map defined by $\chi(L) = \chi(L, 0)$ (that means the result of the 0-surgery on S^3 along L). Then it is known that χ is surjective and A. Kawauchi defined a map $\alpha : \mathbf{M} \to \mathbf{L}^p$ by $\alpha(M) = \min\{L \in \chi^{-1}(M) : \ L' \in \chi^{-1}(M), \pi_1(E(L)) = \pi_1(E(L')) \Rightarrow L < L'\}$ for $M \in \mathbf{M}$, where $E(L)$ is the exterior of L. By using α, we consider $\mathbf{M}$ as a subset of $\mathbf{L}^p$ and introduce the well-order into $\mathbf{M}$.

We completed the table of prime links with lengths up to 10, the table of prime link exteriors with lengths up to 10 and the table of closed connected orientable 3-manifolds with lengths up to 9. We will make our manifold table with lengths up to 10.

References

[K] A. Kawauchi, A tabulation of 3-manifolds via Dehn surgery, Boletin de la Sociedad Matematica Mexicana (3) 10 (2004), 279–304.

[KT1] A. Kawauchi and I. Tayama, Enumerating prime links by a canonical order, Journal of Knot Theory and Its Ramifications Vol. 15, No. 2 (2006) 217–237