Summary of my research **Zengo Tsuboi**

We studied foundation of the decomposition theory of exponential operators. We proposed [1] [1] a general scheme to construct independent determining equations for the relevant decomposition parameters by using Lyndon words on a free Lie algebra.

We studied an analytic Bethe ansatz (ABA) and the T-system (a system of functional relations among transfer matrices) of solvable lattice models in statistical mechanics. At first, we gave solutions of T-system for $U_q(D_r^{(1)})$ [2] and discretized affine Toda field equations [3] from the point of view of ABA.

I proposed [4,5,6] a general theory on ABA based on the Bethe ansatz equations associated with the simple root systems of the Lie superalgebra $sl(r|s)$. A remarkable point is that I did not consider eigenvalue formulae of transfer matrices (DVF) for each model associated with superalgebra separately, rather I considered a wide class of models labeled by the Young superdiagrams, and then proposed a general formula of DVF for it. I also proposed T-system for the DVF. I have generalized the above result to more general superalgebras [7,8]. The above result is the first attempt to establish a systematic theory on ABA for superalgebra. It will also be useful in research on condensed matter physics and integrable spin chains on AdS/CFT in particle physics.

Next, we derived [9,10,11,15] TBA equations for the $osp(1|2s)$ spin chain based on my supersymmetric T-system and the quantum transfer matrix method.

Around 1999, we have researched on fermionic formulae and the general Q-system related to completeness of the Bethe ansatz, and proposed many new formulae [12,13,14]. I also proposed [16] difference L-operators related to the q-characters associated with the twisted quantum affine algebras, and constructed a Casorati determinant solution to T-system. These contribute to a development of the representation theory in mathematics from a point of view of physics.

In general, the TBA equations are infinite number of coupled nonlinear integral equations (NLIE) with an infinite number of unknown functions. Then I have systematically derived NLIE with a finite number (= rank) of unknown functions, which is equivalent to the TBA equations, associated with Lie algebras of arbitrary rank [17,18,20,21,23]. These are not only interesting in theoretical sense but also important for applications to numerical calculations of physical quantities. In fact, they are applicable to calculations of thermodynamics of spin ladder systems. We had an excellent agreement with experimental results in collaboration with Prof. Batchelor et. al. at ANU [19,25].

The evaluation of the correlation functions is a very difficult problem, even in the case of solvable lattice models. We have succeeded to evaluate correlation functions at finite temperatures based on the high temperature expansion technique [22,24].

Prof. Bazhanov (ANU) and I have constructed [26] the Baxter Q-operators for $U_q(\hat{sl}(2|1))$ based on infinite dimensional representations of q-oscillator algebras. We proposed Wronskian-type expressions of T-operators in terms of the Baxter Q-operators and functional relations among the Baxter Q-operators. Our formulation can be applicable to both lattice models and CFT since it is independent of the space where the operators act on. We also showed that connection coefficients of some differential equations satisfy functional relations which are same as the ones for the Baxter Q-operators.

I proposed [27] functional relations among Baxter Q-functions and Wronskian-type solutions of the T-system for $U_q(\hat{gl}(M|N))$. They are yet another form of the so-called q-(super)characters. Moreover, I proved that the Wronskian-like formulae also explicitly solve the functional relations for Bäcklund flows proposed by Kazakov, Sorin and Zabrodin. In [27], I identified that the number of the Baxter Q-operators is 2^{M+N} (two of them are usually 'trivial') for the first time.

Prof. Kazakov (LPTENS), Dr. Gromov (DESY) and I investigated [28] the T-system for AdS/CFT around the strong coupling limit of 't Hooft coupling, where it reduces to the Q-system. We obtained a solution of the Q-system and identified it with a certain supercharacter of an infinite dimensional representation of $gl(4|4)$. We find that the supertrace of monodromy matrix of classical $AdS_5 \times S^5$ superstring coincides with this solution of the Q-system.

Next, we considered the T-system for AdS/CFT for any value of the 't Hooft coupling, and obtained [29] a Wronskian-like solution based on a technique developed in [27].

Prof. Kazakov, Mr. Leurent (LPTENS) and I proposed [30] a new definition of the Baxter Q-operators based on 'co-derivative' on generating functions of supercharacters of $gl(M|N)$, and derived the Bethe ansatz equations without using the Bethe ansatz. We also clarified relations between Bäcklund transformations in the soliton theory and the Baxter Q-operators on the level of operator.

[1] See my publication list for the reference number of each paper.