Research programs

(1) **Relation between the center of mass and asymptotic behavior of solution to diffusion equations**

In my previous works (4) it is suggested that there is a deep relation between the center of mass and asymptotic behavior of solution to diffusion equations in the whole space. However most part of the works in this direction do not take into account the center of mass of solution. For example, see the cases of generalized Burgers equations (M. Kato, Osaka Math. J., 44 (2007) 923–943) and Porous-media equations (A. Friedman and S. Kamin, Trans. Amer. Math. Soc. 262 (1980) 551–563, etc). The aim is here to consider generalized Burgers equations and Porous-media equations in the whole space, and to observe a relation between the center of mass and asymptotic behavior of solution.

(2) **Drift-diffusion system with slowly decaying initial data in the whole space**

There are a lot of works on the asymptotic behavior of global solutions to drift-diffusion equations in the whole space when the initial data is in L^1 and is small enough. On the other hand, in the case where the initial data is bounded, but not necessary in L^1, Nishihara and Narazaki proved the existence and asymptotic profile of solutions to the semilinear heat equation. However it seems that the works based on this view point are little-known. The purpose is here to investigate the existence and asymptotic profile of solutions to drift-diffusion system (for example, chemotaxis model) with slowly decaying initial data in the whole space, and to observe how the drift term and decay of initial data at space infinity influence the asymptotic behavior of solutions.

(3) **A parabolic-elliptic system modeling chemotaxis in two dimensional space**

Nagai and Senba considered a parabolic-elliptic system that is a simplified model of chemotaxis with a sensitivity function $\psi(v)$ specified as $\psi(v) = v^p$ $(p > 0)$ and $\psi(v) = \log v$ discussed whether the blow-up of nonnegative solutions can occur or not in radially symmetric situations (T. Nagai and T. Senba, Adv. Math. Sci. Appl. 8 (1998) 145–156). However there are not works analyzed the above system without assuming radially symmetric situations as far as I know. The aim is here to treat a parabolic-elliptic system of chemotaxis with the above sensitivity function $\psi(v)$ in $\mathbb{R}^2$ without assuming radially symmetric situations, and to study the following problems:

(a) the global existence and blow-up of nonnegative solutions.

(b) the asymptotic behavior of nonnegative global solutions.

Furthermore it is clear how sensitivity functions $\psi(v)$ influence the structure of solutions.