Result

In quantum mechanics, boundary conditions are considered when the domain of wave functions has boundary or potential is given by generalized function. Boundary conditions are discussed in the context of self-adjoint extension in functional analysis [1]. Although the Dirichlet boundary condition was employed in many cases, there exists an infinitely number of boundary conditions. Recently, how non-Dirichlet boundary conditions affect the physics is studied in various physical fields.

I have mainly studied effects of the boundary conditions in quantum many body system. Firstly, I have studied boundary effects of 3 body Calogero model. Concerning previous work [2] where a class of boundary problems has been studied, we imposed a different class of boundary conditions on wave functions and solve it. Although the boundary conditions employed in [2] have symmetry with respect to the classical Hamiltonian, the spectrum conditions are given by the transcendental equations. Although the boundary conditions employed in our research [3] break the symmetry which classical Hamiltonian possesses, we obtained explicit spectra and confirm a kind of periodicity of a spectrum. The second research is on thermodynamic equilibrium state of boundary systems. In the previous work [4,5], Fülöp et al. have considered a one-dimensional box with two Dirichlet conditions and inserted thick partition that has imposed the Dirichlet condition at one side and the Neumann condition at the other side. N free particles were put in the two chambers divided by the partition, respectively. Due to the inserted partition, the pressure of left chamber is different from the pressure of right chamber. In [4,5], the temperature dependence of the pressure difference was obtained by analytical and numerical calculations. I has already taken a step forward, generalized the boundary condition of the box, and obtained equations of state of the system [6]. I have shown that obtained equations are shifted from equation of state of ideal gas and boundaries act as van der Waals force in classical statistical mechanics. Thirdly, I generalize Lieb-Liniger model [7], which is N body model on the circle and its particles interact with each other by the delta functions. Lieb-Liniger A model is a multi-object model on the circumference, and is a system in which particles carry out an interaction with a delta function. In particular, the model has been realized in recent years using the optical lattice [8]. The particles were assumed to be bosons in Lieb-Liniger model. I develop general theory of distribution that is consistent with anyon and propose the anyonic extension of Lieb-Liniger model according to the general theory. Moreover, I solve the generalized model by using Bethe ansatz. My model includes generalization of Lieb-Liniger model, [7, 9, 10], etc. Fourthly, I have calculated approximate energy spectrum of the generalized Lieb-Liniger model in the attractive potential region and obtain equation of state of the system. The equation of state is similar to that of van der Waals equation of state. The second and fourth study show that boundary conditions, in thermodynamic equilibrium state, lead to similar effect that derives from the van der Waals force in classical mechanics.

Besides the boundary condition, I have also studied quantum information in [11]. This study points out that the substitution symmetry of particles plays an important role in the classification of quantum entanglement. In addition, I also study AGT relation. Using the matrix model related to AGT relation proposed by [12], we calculate the scaling limit and confirm that the result coincides with the corresponding Nekrasov's partition function[13].

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