Research plan

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1. An algebra is said to be of *tame type* if all but only finitely many isoclasses of indecomposable modules with the same dimension are controlled by only finitely many parameters. Note that any representation-finite algebra is of tame type by the 2nd Brauer-Thrall conjecture. Thus the following conjecture should be considered.

   Conjecture. *Any selfinjective algebra of tame type has stable dimension at most* 1.

   In general, the converse does not hold. Note that any selfinjective algebra having representation dimension 3 has stable dimension 1, and hence there is some evidence obtained for this conjecture. There are subclasses of algebras of tame type such as *domestic* algebras and algebras of *polynomial growth*. The inclusions are given as follows: $\{\text{representation-finite}\} \subset \{\text{domestic}\} \subset \{\text{polynomial growth}\} \subset \{\text{tame type}\}$. The stable dimension of a standard selfinjective algebra is related to the derived dimension of an algebra with finite global dimension by a covering technique. Actually, any non-representation-finite domestic standard selfinjective algebra and any non-domestic standard selfinjective algebra of polynomial growth have stable dimension 1 since any hereditary algebra of Euclidean type and any (canonical) tubular algebra have derived dimension 1, respectively. Thus, we will first consider the conjecture for standard selfinjective algebras of tame type but not of polynomial growth. We also consider the conjecture for non-standard selfinjective algebras.
2. Second, in connection with the conjecture above, we will consider the natural question whether the derived dimension of wild canonical algebras is 2. Such algebras have global dimension at most 2. Note that any canonical tubular algebra has derived dimension 1 (Oppermann). In addition, we will investigate the stable dimension of selfinjective algebras given by canonical ones. Since any algebra having global dimension at most 2 is quasi-hereditary, we will also consider the derived dimension of quasi-hereditary algebras.
3. According to Rouquier, an exterior algebra $\wedge(k^n)$ has representation dimension $n+1$, derived dimension n and stable dimension $n-1$. On the other hand, any representation-finite selfinjective algebra has representation dimension 2, derived dimension 1 and stable dimension 0 (Auslander, Chen-Ye-Zhang and Han). Therefore, we have two natural questions.

   Question 1. Is the difference between the representation dimension and the derived dimension at least 1 ?

   Question 2. Is the difference between the derived dimension and the stable dimension at least 1 ?

   At present, it is not known whether there exists an (selfinjective) algebra such that these dimensions coincide. Oppermann also pointed out them. Thus, the third purpose is to investigate these question.