Abstract of the results of my research

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The aim of representation theory of algebras is to investigate the structure of their module categories. So far I mainly have been studying the derived categories, the equivalences between them and some invariants under derived equivalences. They are useful tools in representation theory of algebras. For example, the important information of algebras such as the Grothendieck group and the finiteness of the global dimension is invariant under derived equivalences. Recently, it turns out that they play important roles in Lie theory and noncommutative algebraic geometry.

By the way, Rickard's works are most important to study them: Rickard's theorem is a generalization of Morita's theorem on Morita equivalences. Namely, Rickard generalized the concept of progenerators to that of tilting complexes: The progenerators define Morita equivalences and the tilting complexes define derived equivalences.

On the other hand, there is the concept of tilting modules, which is also a generalization of that of progenerators. An algebra A and the endomorphism ring $B = \mathrm{End}_A(T)$ of a tilting A-module T are not Morita equivalent but some subcategories of their module categories are equivalent. Moreover, A has finite global dimension if and only if so does B. So, tilting theory must be important. It is clear that as stalk complexes, tilting modules are tilting complexes. Therefore, Morita theory for derived categories is a generalization of tilting theory. The relationship between them became clearer because I showed that tilting stalk complexes give conversely tilting modules [1].

By the way, for a selfinjective algebra, the representation dimension is invariant under derived equivalences (Guo+Rickard). This was proposed by Auslander based on an idea that the representation-theoretic properties such as representation-finiteness must be controlled by the homological invariants such as the global dimension. This concept was not handled easily. The representation dimension is, however, important because it has provided many interesting issues in representation theory of algebras and has become one of the origins of cluster tilting theory proposed recently.

An important concept introduced recently in connection with the representation dimension is Rouquier's dimension of triangulated category. This is an attempt to formulate in category theory the geometric 'dimension' as the global dimension is. The stable dimension and the derived dimension of (selfinjective) algebras are closely related to the global dimension and the representation dimension (Rouquier) so that they are especially interesting in representation theory of algebras.

The *stable dimension* of a selfinjective algebra is the dimension of its stable module category as a triangulated category. This is invariant under derived equivalences (Rickard). By definition, if a selfinjective algebra is representation-finite, then it has stable dimension zero. Then a natural question arises as to whether the converse should also hold. I have given an affirmative answer to this [3]. Although this was expected to hold by some experts, it had not been proved before.

As an application, I also exhibit in [5] (or [4]) that algebras having derived dimension zero are closely related to selfinjective ones having stable dimension zero, improving Chen-Ye-Zhang's result. In addition, I propose a new conjecture related to a conjecture for the representation dimension. After this, I will consider this conjecture.