

Research program

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- **Symplectic structures on small covers and real moment-angle manifolds**

A small cover of dimension n is a closed manifold on which $(\mathbb{Z}_2)^n$ acts effectively and whose orbit space is a simple convex polytope. Among small covers, there are many aspherical manifolds. As well as toric manifolds, each small cover corresponds to a combinatorial object which is a so-called characteristic pair. A necessary and sufficient condition is known for a small cover to be orientable. The author gave a necessary and sufficient condition for a real Bott manifold, which is a small cover over a cube, to have a symplectic structure. But, we do not know for other small covers to have a symplectic structure. The purpose of this study is to characterize all small covers to have a symplectic structure. It is very interesting even the case when the orbit space is a product of some polygons. The author also wants to work on real moment angle manifolds, which are finite covering of small covers.

- **Manifolds with actions of tori**

To a torus manifold, we can assign a multi-fan introduced by A. Hattori and M. Masuda. The multi-fan associated with a torus manifold allows us to compute some invariants. To a GKM space, we can assign a GKM graph which is a labeled graph. The GKM graph allows us to compute its cohomology ring. Similarly to torus manifolds and GKM spaces, the author wants to assign a combinatorial object to a manifold on which a torus acts under a special condition in order to study torus actions. So the author would like to consider generalization and some special cases, that is,

1. the case when the dimension of the torus acting on the manifold is “small”,
2. the case when the action of the torus preserves a certain geometric structure.

It would be worth to describe such manifolds with combinatorial objects. Moreover, it would be interested if we could find an application to combinatorics.