Study results

Hiroaki Ishida

- **Characterization of real Bott manifolds which admit a symplectic structure**

  Finding a necessary and sufficient condition for a closed manifold to have a symplectic structure is a fundamental problem. So far, some necessary conditions are known, but we do not know a complete characterization for a closed manifold to have a symplectic structure. The author gave a necessary and sufficient condition for a real Bott manifold, which is the total space of a certain iterated S^1-bundle, to have a symplectic structure ([1], [4]).

- **Classification of Bott towers**

  A Bott tower is an iterated $\mathbb{C}P^1$-bundle such that each fibration is the projectivization of the Whitney sum of two complex line bundles. The total space of a Bott tower is called a Bott manifold. M. Masuda and D. Y. Suh conjectured that Bott manifolds are distinguished by their integral cohomology rings (cohomological rigidity problem). The author showed that Bott towers are distinguished by their filtered cohomology rings ([2], [5]). Namely, it says that the cohomological rigidity problem holds in a special case.

- **Generalization of toric manifolds**

  As a topological analogue, a quasitoric manifold introduced by M. Davis and T. Junuskiewicz is well-studied. Y. Fukukawa, M. Masuda and I introduced the notion of topological toric manifolds, which is another topological analogue with a different view point ([6]). I showed that if there is an in variant stably complex structure on a topological toric manifold, then it is a toric manifold ([7]).

- **Characterization of complex torus manifolds and relation with toric varieties**

  A torus manifold of dimension $2n$ is a closed oriented manifold on which $(S^1)^n$ acts effectively and having a fixed point. A complex torus manifold of complex dimension n is a closed complex manifold on which $(S^1)^n$ acts effectively , as biholomorphisms and having a fixed point. M. Masuda and I determined the Todd genus of a complex torus manifold whose odd-degree cohomology groups vanish ([9]). After that, Y. Karshon and I showed that any complex torus manifold is equivariantly biholomorphic to a toric manifold which is a non-singular complete toric variety ([10]). Namely, we have a characterization of toric manifolds among torus manifolds.